

具有未知死区的 SISO 非仿射非线性系统 间接自适应模糊控制

周卫东¹, 廖成毅¹, 郑 兰¹, 程 华²

(1. 哈尔滨工程大学 自动化学院, 150001 哈尔滨; 2. 齐齐哈尔市建华机械有限公司, 161006 黑龙江 齐齐哈尔)

摘 要: 为了解决含有未知死区输入特性的 SISO 非仿射非线性系统的跟踪控制问题, 提出了基于模糊自适应方法的控制器设计方案, 把未知死区分解为一个线性项和一个扰动类似项, 当系统状态可测时, 利用模糊逻辑系统设计间接自适应模糊控制器, 并结合跟踪误差信息设计自适应律; 系统状态不可测时, 通过引入误差观测器估计的状态变量, 设计了间接自适应模糊输出反馈控制器。理论论证过程说明两种控制器能使系统的跟踪误差最终收敛到零的某一邻域, 且闭环系统所有信号均有界。仿真实验结果表明利用所设计控制方案可以使系统完成跟踪控制任务。

关键词: 未知死区; 非仿射非线性系统; 自适应模糊控制; 误差观测器; 输出反馈

中图分类号: TP273

文献标志码: A

文章编号: 0367-6234(2014)10-0110-07

Indirect adaptive fuzzy control for SISO nonaffine nonlinear system with unknown dead-zone input

ZHOU Weidong¹, LIAO Chengyi¹, ZHENG Lan¹, CHENG Hua²

(1. College of Automation, Harbin Engineering University, 150001 Harbin, China;

2. Qiqihaer Jianhua Machinery Co., Ltd., 161006 Qiqihaer, Heilongjiang, China)

Abstract: To cope with the controller design problem that the control input of single-input single-output (SISO) nonaffine nonlinear system is with an unknown dead-zone character, a controller design strategy based on fuzzy adaptive technique is proposed. Within this scheme, the unknown dead-zone is treated as a combination of a linear and a bounded disturbance-like term. For the case that the system states are measurable, an indirect adaptive fuzzy controller is constructed by using fuzzy logic systems and adaptive laws are given out according to the information of tracking error. On basis of the above controller design method, another indirect adaptive fuzzy output-feedback controller for immeasurable system states is designed based on the estimated states which are generated by an error observer. The theoretic prove process indicates that the two controllers can guarantee that the tracking errors converge to a small neighbourhood of the origin and all signals in closed-loop system are bounded. Simulation results demonstrate that the constructed controllers can make the system achieve the tracking control objective.

Keywords: unknown dead-zone; nonaffine nonlinear system; adaptive fuzzy control; error observer; output-feedback

在工业过程中经常遇到死区现象, 其非线性

特性会严重影响控制系统的性能, 甚至可能导致系统不稳定。因此, 许多学者对具有未知死区非线性输入的系统作了大量研究, 并取得了一定的成果^[1-13]。文献[1-2]针对一类具有死区环节的非线性系统提出构造死区反函数来补偿死区环节对系统带来的影响, 但是该方法中不光滑的反函数

收稿日期: 2013-09-16.

基金项目: 国家自然科学基金资助项目(61102107, 61374208)。

作者简介: 周卫东(1966—), 男, 博士生导师;

廖成毅(1986—), 男, 博士研究生。

通信作者: 廖成毅, cugliaocy@163.com.

可能会导致系统发生抖振现象. Zhou J. 等通过设计光滑的死区反函数避免了抖振问题^[3-4]. 文献 [5-6] 分别针对运动控制系统和直流电机系统提出了模糊设计方法, 即根据专家经验设计基于模糊逻辑系统的死区补偿器, 利用模糊方法构造控制器, 然而该方案会过度地依赖于专家经验, 当找不到合适的描述死区非线性特性的模糊规则时, 不能利用这种方法处理死区问题. 在文献 [7-9, 11-13] 中, 死区被分解为一个线性项和一个有界的扰动类似项, 然后利用鲁棒控制技术设计控制器, 取得了理想的控制效果. 上述研究均是针对仿射非线性系统, 而大多数实际系统如飞控系统、化学反应控制系统等都是非仿射系统, 近年来非仿射非线性系统的研究取得了很多成果^[14-17], 遗憾的是针对非仿射非线性系统的研究, 几乎没有人考虑含有死区输入的情况.

本文针对一类具有未知死区输入环节的 SISO 非仿射非线性系统, 给出了一种间接自适应模糊控制方法. 当系统状态可测时, 采用模糊逻辑系统逼近系统中的非线性函数, 构造了间接自适应模糊控制器. 当系统状态不可测时, 设计观测器估计跟踪误差, 基于估计得到的状态设计间接自适应模糊输出反馈控制器. 最后利用 Lyapunov 稳定性定理证明了跟踪误差收敛到零的某一邻域, 同时保证闭环系统所有信号有界.

1 问题描述

考虑如下一类 SISO 非仿射系统

$$\begin{cases} \dot{x}_i = x_{i+1}, \\ \dot{x}_n = F(\mathbf{x}, D(u)) + d(t), \\ y = x_1. \end{cases} \quad (1)$$

其中: $u \in \mathbf{R}$ 为系统输入量, $y \in \mathbf{R}$ 为系统输出量, $\mathbf{x} = [x_1, \dots, x_n]^T \in \mathbf{R}^n$ 为系统的状态向量, $d(t)$ 为外界干扰且存在常数 $\bar{d}$ 满足 $|d(t)| \leq \bar{d}$, $F(\mathbf{x}, D(u))$ 为光滑的未知函数, $D(u)$ 为参数未知的死区输入.

控制目的: 设计控制器使得系统输出 y 跟踪指定期望轨迹 y_d , 同时闭环系统所有信号均有界.

假设 1 对于任意 $\mathbf{x} \in \Omega_x \subset \mathbf{R}^n$, 存在大于零的常数 $\underline{g}, \bar{g}$ 使得函数 $\mathbf{g}(\mathbf{x}) = \frac{\partial F(\mathbf{x}, D(u))}{\partial D(u)}$, 满足条件 $\underline{g} < \mathbf{g}(\mathbf{x}) \leq \bar{g}$ 成立, 其中 Ω_x 为系统的可控域.

假设 2 $y_d(t)$ 及其对时间导数 $y_d^{(i)}(t)$ 为有界的光滑函数, $i = 1, 2, \dots, n$.

假设 3 对于任意 $\mathbf{x} \in \Omega_x \subset \mathbf{R}^n$ 存在常数 g_r 使得 $|\dot{\mathbf{g}}(\mathbf{x})| \leq g_r$ 成立.

输入为 u 输出为 $D(u)$ 的死区模型描为

$$D(u) = \begin{cases} m_r(u(t) - b_r), & u(t) \geq b_r; \\ 0, & -b_l < u(t) < b_r; \\ m_l(u(t) + b_l), & u(t) \leq -b_l. \end{cases}$$

死区模型满足条件: 1) 死区输出 $D(u)$ 不可测; 2) 死区参数未知, 满足 $0 \leq b_r \leq \bar{b}_r, 0 \leq b_l \leq \bar{b}_l, 0 < m_l \leq \bar{m}_l, 0 < m_r \leq \bar{m}_r$.

将未知死区转化为

$$D(u) = m(t)u + \beta(t). \quad (2)$$

其中: $m(t) \triangleq \begin{cases} m_l, & u \leq 0; \\ m_r, & u > 0. \end{cases}$

$$\beta(t) \triangleq \begin{cases} -m_r b_r, & u \geq b_r; \\ -m(t)u, & -b_l < u < b_r; \\ m_l b_l, & u \leq -b_l. \end{cases}$$

根据条件 2 可知存在大于零的常数 $\beta = \max(\bar{m}_r \bar{b}_r, \bar{m}_l \bar{b}_l)$, 使得 $|\beta(t)| \leq \beta$.

2 自适应模糊控制器设计

首先利用中值定理将非仿射非线性形式转化为仿射非线性形式, 有

$$F(\mathbf{x}, D(u)) = F(\mathbf{x}, 0) + \frac{\partial F(\mathbf{x}, D(u))}{\partial D(u)} \Big|_{D(u)=\lambda} (D(u) - 0) = f(\mathbf{x}) + g(\mathbf{x})D(u). \quad (3)$$

其中: $f(\mathbf{x}) = F(\mathbf{x}, 0), \lambda \in [0, D(u)]$,

$$g(\mathbf{x}) = \frac{\partial F(\mathbf{x}, D(u))}{\partial D(u)} \Big|_{D(u)=\lambda}.$$

结合式 (2), 系统 (1) 可转换为

$$y^{(n)} = f(\mathbf{x}) + g(\mathbf{x})mu + d_0. \quad (4)$$

其中: $d_0 = g(\mathbf{x})\beta(t) + d(t)$, 则存在大于零的常数 $\bar{d}_0$ 使得 $|d_0| \leq \bar{d}_0$.

针对经转化得到的仿射非线性系统, 分系统状态可测与不可测两种情况进行分析.

2.1 系统状态可测

因为 $f(\mathbf{x})$ 和 $g(\mathbf{x})$ 为未知光滑函数, 利用模

糊逻辑系统 $\hat{f}(\mathbf{x} \mid \hat{\theta}_f) = \hat{\theta}_f^T \xi(\mathbf{x})$ 和 $\hat{g}(\mathbf{x} \mid \hat{\theta}_g) = \hat{\theta}_g^T \xi(\mathbf{x})$ 来分别逼近 $f(\mathbf{x})$ 和 $g(\mathbf{x})$. 其中 $\hat{\theta}_f = [\hat{\theta}_{f1}, \hat{\theta}_{f2}, \dots, \hat{\theta}_{fM}]^T, \hat{\theta}_g = [\hat{\theta}_{g1}, \hat{\theta}_{g2}, \dots, \hat{\theta}_{gM}]^T$ 均为自适应参数. $\xi(\mathbf{x}) = [\xi_1(\mathbf{x}), \xi_2(\mathbf{x}), \dots, \xi_M(\mathbf{x})]^T$ 为模糊系统的基函数. 式中 $\xi_j(\mathbf{x}) \triangleq$

$$\frac{\prod_{i=1}^n \mu_{F_i}(x_i)}{\sum_{l=1}^M \left(\prod_{i=1}^n \mu_{F_i}(x_i) \right)}, \quad j = 1, 2, \dots, M.$$

根据模糊逻辑系统的万能逼近特性^[18],存在最优逼近参数 θ_f^* 和 θ_g^* 分别满足

$$\theta_f^* = \arg \min_{\theta_f} (\sup_{x \in x} |\hat{f}(x) - f(x)|),$$

$$\theta_g^* = \arg \min_{\theta_g} (\sup_{x \in x} |\hat{g}(x) - g(x)|).$$

估计误差为

$$\tilde{\theta}_f = \theta_f^* - \hat{\theta}_f, \tilde{\theta}_g = \theta_g^* - \hat{\theta}_g.$$

定义最优逼近误差为

$$\omega = f(x) - \hat{f}(x | \theta_f^*) + [g(x) - \hat{g}(x | \theta_g^*)]u = \varepsilon_f + \varepsilon_g u.$$

根据万能逼近特性可知,选择适当的模糊规则数,可以保证逼近误差任意小,因此存在未知常数 $\bar{\varepsilon}$ 使得 $|\varepsilon_f + \varepsilon_g u| \leq \bar{\varepsilon}$.

定义跟踪误差 $e = y_d - y$, 则误差向量 $e = [e, \dot{e}, \dots, e^{(n-1)}]^T$.

由式(3)有系统跟踪误差方程为

$$\dot{e} = (A - BK^T)e + B[y_d^{(n)} + K^T e - f(x) - g(x)mu - d_0]. \quad (5)$$

式中: $B = [0 \quad \dots \quad 0 \quad 1]^T$,

$$A = \begin{bmatrix} \mathbf{0}_{(n-1) \times 1} & \mathbf{I}_{(n-1) \times (n-1)} \\ 0 & \mathbf{0}_{1 \times (n-1)} \end{bmatrix}.$$

选择向量 $K = [k_n, k_{n-1}, \dots, k_1]^T$ 使得 $s^n + k_1 s^{n-1} + k_2 s^{n-2} + \dots + k_n$ 为 Hurwitz 多项式. 因此 $A_1 = A - BK^T$ 为稳定的矩阵,即给定一个正定对称矩阵 P ,存在正定矩阵 Q 满足 Lyapunov 方程

$$A_1^T P + P A_1 = -Q. \quad (6)$$

定理1 对于系统(1),采用如下控制律与自适应律,可以使得系统输出跟踪给定期望信号,同时保证闭环系统所有信号均有界.

控制律为

$$u = \hat{\alpha}v, \quad (7)$$

$$v = \frac{1}{\hat{g}(x)}(-\hat{f}(x) + y_d^{(n)} + K^T e - \hat{\rho}). \quad (8)$$

自适应律为

$$\dot{\hat{\theta}}_f = -\gamma_1 e^T P B \xi(x) - \eta_1 \hat{\theta}_f, \quad (9)$$

$$\dot{\hat{\theta}}_g = -\gamma_2 e^T P B \xi(x)v - \eta_2 \hat{\theta}_g, \quad (10)$$

$$\dot{\hat{\alpha}} = \gamma_3 e^T P B v - \eta_3 \hat{\alpha}, \quad (11)$$

$$\dot{\hat{\rho}} = -\gamma_4 e^T P B - \eta_4 \hat{\rho}. \quad (12)$$

其中: $\hat{\alpha}$ 为 $\alpha = \frac{1}{m}$ 的估计, $\hat{\rho}$ 为 $\rho = \bar{\varepsilon} + \bar{d}_0$ 的估计,

$\gamma_1, \gamma_2, \gamma_3, \gamma_4, \eta_1, \eta_2, \eta_3, \eta_4$ 均为大于零的常数.

证明 取如下 Lyapunov 候选函数

$$V = \frac{1}{2} e^T P e + \frac{1}{2\gamma_1} \tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\gamma_2} \tilde{\theta}_g^T \tilde{\theta}_g +$$

$$\frac{mg(x)}{2\gamma_3} \tilde{\alpha}^2 + \frac{1}{2\gamma_4} \tilde{\rho}^2, \quad (13)$$

对 V 求时间的导数并结合(5)、(6)可得

$$\begin{aligned} \dot{V} = & -\frac{1}{2} e^T Q e + e^T P B [y_d^{(n)} + K^T e - f(x) - \\ & g(x)mu - d_0] - \frac{1}{\gamma_1} \tilde{\theta}_f^T \dot{\tilde{\theta}}_f - \frac{1}{\gamma_2} \tilde{\theta}_g^T \dot{\tilde{\theta}}_g + \\ & \frac{mg(x)}{2\gamma_3} \tilde{\alpha}^2 - \frac{mg(x)}{\gamma_3} \tilde{\alpha} \dot{\tilde{\alpha}} - \frac{1}{\gamma_4} \tilde{\rho} \dot{\tilde{\rho}}. \end{aligned} \quad (14)$$

其中: $\tilde{\alpha}, \tilde{\rho}$ 分别为 α, ρ 估计误差, 定义为 $\tilde{\alpha} = \alpha - \hat{\alpha}$.

将控制律式(7)、(8)以及自适应律式(9)~(12)代入式(14)整理可得

$$\begin{aligned} \dot{V} = & -\frac{1}{2} e^T Q e + \frac{\eta_1}{\gamma_1} \tilde{\theta}_f^T \hat{\theta}_f + \frac{\eta_2}{\gamma_2} \tilde{\theta}_g^T \hat{\theta}_g + \\ & \frac{\eta_3 mg(x)}{\gamma_3} \tilde{\alpha} \hat{\alpha} + \frac{\eta_4}{\gamma_4} \tilde{\rho} \hat{\rho} + \frac{m \dot{g}(x)}{2\gamma_3} \tilde{\alpha}^2. \end{aligned} \quad (15)$$

根据假设1和假设3,利用不等式性质整理式(15)有

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2} e^T Q e - \frac{\eta_1}{2\gamma_1} \tilde{\theta}_f^T \tilde{\theta}_f - \frac{\eta_2}{2\gamma_2} \tilde{\theta}_g^T \tilde{\theta}_g - \\ & \left(\eta_3 - \frac{g_r}{g} \right) \frac{mg(x)}{2\gamma_3} \tilde{\alpha}^2 - \frac{\eta_4}{2\gamma_4} \tilde{\rho}^2 + \frac{\eta_4}{2\gamma_4} \rho^2 + \\ & \frac{\eta_3 m \bar{g}}{\gamma_3} \alpha^2 + \frac{\eta_2}{2\gamma_2} \theta_g^{*T} \theta_g^* + \frac{\eta_1}{2\gamma_1} \theta_f^{*T} \theta_f^* \leq \\ & -\mu_0 V + C_0. \end{aligned} \quad (16)$$

其中: $\mu_0 = \min \left(\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \eta_1, \eta_2, \eta_3 - \frac{g_r}{g}, \eta_4 \right)$,

$$C_0 = \frac{\eta_1}{2\gamma_1} \theta_f^{*T} \theta_f^* + \frac{\eta_2}{2\gamma_2} \theta_g^{*T} \theta_g^* + \frac{\eta_3 m \bar{g}}{\gamma_3} \alpha^2 + \frac{\eta_4}{2\gamma_4} \rho^2, \text{ 选取 } \eta_3 \text{ 使得 } \eta_3 - \frac{g_r}{g} > 0.$$

由式(16)可得

$$V(t) \leq \left[V(0) - \frac{C_0}{\mu_0} \right] e^{-\mu_0 t} + \frac{C_0}{\mu_0}. \quad (17)$$

由式(17)有如下结论: 存在 T 使得当 $t \geq T$

时有跟踪误差 $e \leq \sqrt{\frac{2C_0}{\lambda_{\min}(P)\mu_0}}$ 成立,选取合适的

参数可使得跟踪误差任意小,实现跟踪任务,同时根据 $V(t)$ 的定义可知闭环系统所有信号均有界.定理1证毕.

2.2 系统状态不可测

在实际工程中,通常只有系统输出可测,而系统状态通常是不可测量的,即 y 可测, $\mathbf{x} = [x_2, \dots, x_n]^T$ 不可测.此时只能利用状态的估计 $\hat{\mathbf{x}} = [\hat{x}_1,$

$\dots, \hat{x}_n]^T$ 设计间接自适应模糊控制器.

跟踪误差向量为 $\mathbf{e} = [e, \dot{e}, \dots, e^{(n-1)}]^T$, 定义

误差的估计为 $\hat{\mathbf{e}} = [\hat{e}, \dot{\hat{e}}, \dots, \hat{e}^{(n-1)}]^T$, 即构造误差观测器估计误差向量, 则观测误差为 $\tilde{\mathbf{e}} = \mathbf{e} - \hat{\mathbf{e}}$. 考虑到系统状态不可测, 用 $\hat{f}(\hat{\mathbf{x}} | \hat{\boldsymbol{\theta}}_f) = \hat{\boldsymbol{\theta}}_f^T \xi(\hat{\mathbf{x}})$ 和 $\hat{g}(\hat{\mathbf{x}} | \hat{\boldsymbol{\theta}}_g) = \hat{\boldsymbol{\theta}}_g^T \xi(\hat{\mathbf{x}})$ 来分别逼近未知函数 $f(\mathbf{x})$ 和 $g(\mathbf{x})$. 同理, 存在最优逼近满足

$$\boldsymbol{\theta}_f^* = \arg \min_{\boldsymbol{\theta}_f} (\sup_{\mathbf{x} \in \mathbb{X}} |\hat{f}(\hat{\mathbf{x}}) - f(\mathbf{x})|),$$

$$\boldsymbol{\theta}_g^* = \arg \min_{\boldsymbol{\theta}_g} (\sup_{\mathbf{x} \in \mathbb{X}} |\hat{g}(\hat{\mathbf{x}}) - g(\mathbf{x})|).$$

参数误差为

$$\tilde{\boldsymbol{\theta}}_f = \boldsymbol{\theta}_f^* - \hat{\boldsymbol{\theta}}_f, \quad \tilde{\boldsymbol{\theta}}_g = \boldsymbol{\theta}_g^* - \hat{\boldsymbol{\theta}}_g.$$

定义 $f(\mathbf{x})$ 和 $g(\mathbf{x})$ 的最优逼近误差为

$$\begin{aligned} \sigma = & [f(\mathbf{x}) - \hat{f}(\hat{\mathbf{x}} | \boldsymbol{\theta}_f^*)] + [g(\mathbf{x}) - \\ & \hat{g}(\hat{\mathbf{x}} | \boldsymbol{\theta}_g^*)]u = \{[f(\mathbf{x}) - f(\mathbf{x} | \boldsymbol{\theta}_f^*)] + \\ & [f(\mathbf{x} | \boldsymbol{\theta}_f^*) - \hat{f}(\hat{\mathbf{x}} | \boldsymbol{\theta}_f^*)]\} + \{[g(\mathbf{x}) - \\ & g(\mathbf{x} | \boldsymbol{\theta}_g^*)] + [g(\mathbf{x} | \boldsymbol{\theta}_g^*) - \hat{g}(\hat{\mathbf{x}} | \boldsymbol{\theta}_g^*)]\}u. \end{aligned}$$

则存在未知正常数 $\bar{\sigma}$ 满足 $|\sigma| \leq \bar{\sigma}$.

由式(4)可得系统跟踪误差方程为

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B}[\gamma_d^{(n)} - f(\mathbf{x}) - g(\mathbf{x})mu - d_0]. \quad (18)$$

设计误差观测器为

$$\begin{cases} \dot{\hat{\mathbf{e}}} = \mathbf{A}\hat{\mathbf{e}} - \mathbf{B}\mathbf{K}_0^T \hat{\mathbf{e}} + \mathbf{K}_c(\mathbf{e} - \hat{\mathbf{e}}), \\ \hat{\mathbf{e}} = \mathbf{C}^T \hat{\mathbf{e}}. \end{cases} \quad (19)$$

式中: $\mathbf{C} = [1, 0, \dots, 0]^T$, $\mathbf{K}_0^T = [k_1^0, k_{n-1}^0, \dots, k_1^0]$ 为反馈增益, $\mathbf{K}_c^T = [k_1^c, k_2^c, \dots, k_n^c]$ 为观测器增益, 选择 $\mathbf{K}_0$ 、 $\mathbf{K}_c$ 分别使 $\mathbf{A} - \mathbf{B}\mathbf{K}_0^T$ 、 $\mathbf{A} - \mathbf{K}_c\mathbf{C}^T$ 为稳定矩阵.

假设 4 正定矩阵 $\mathbf{P}_1$ 和 $\mathbf{P}_2$ 分别满足方程

$$(\mathbf{A} - \mathbf{B}\mathbf{K}_0^T)^T \mathbf{P}_2 + \mathbf{P}_2(\mathbf{A} - \mathbf{B}\mathbf{K}_0^T) + \mathbf{P}_2 \mathbf{K}_0 \mathbf{K}_0^T \mathbf{P}_2 = -\mathbf{Q}_2, \quad (20)$$

$$(\mathbf{A} - \mathbf{K}_c\mathbf{C}^T)^T \mathbf{P}_1 + \mathbf{P}_1(\mathbf{A} - \mathbf{K}_c\mathbf{C}^T) + \mathbf{C}\mathbf{C}^T = -\mathbf{Q}_1. \quad (21)$$

由式(18)、(19)可得

$$\begin{aligned} \dot{\hat{\mathbf{e}}} = & (\mathbf{A} - \mathbf{K}_c\mathbf{C}^T) \hat{\mathbf{e}} + \mathbf{B}[\gamma_d^{(n)} + \mathbf{K}_0^T \hat{\mathbf{e}} - \\ & f(\mathbf{x}) - g(\mathbf{x})mu - d_0]. \end{aligned} \quad (22)$$

定理 2 对于系统(1), 当系统状态不可测时, 采用如下控制律和自适应律, 可以使得系统输出跟踪给定期望信号, 同时保证闭环系统所有信号均有界.

控制律

$$u = \hat{\alpha}_2 v_2, \quad (23)$$

$$v_2 = \frac{1}{\hat{g}(\hat{\mathbf{x}})} (\gamma_d^{(n)} + \mathbf{K}_0^T \hat{\mathbf{e}} - \hat{f}(\hat{\mathbf{x}}) - \hat{\rho}_2). \quad (24)$$

自适应律

$$\dot{\hat{\boldsymbol{\theta}}}_f = -\gamma_{21} \|\mathbf{P}_1\|^2 \hat{\boldsymbol{\theta}}_f - \eta_{21} \hat{\boldsymbol{\theta}}_f, \quad (25)$$

$$\dot{\hat{\boldsymbol{\theta}}}_g = -\gamma_{22} \|\mathbf{P}_1\|^2 u^2 \hat{\boldsymbol{\theta}}_g - \eta_{22} \hat{\boldsymbol{\theta}}_g, \quad (26)$$

$$\dot{\hat{\alpha}}_2 = -\gamma_{23} \|\mathbf{P}_1\|^2 \|\hat{\boldsymbol{\theta}}_g\|^2 v_2^2 \hat{\alpha}_2 - \eta_{23} \hat{\alpha}_2, \quad (27)$$

$$\dot{\hat{\rho}}_2 = -\gamma_{24} \|\mathbf{P}_1\|^2 \hat{\rho}_2 - \eta_{24} \hat{\rho}_2. \quad (28)$$

其中: $\hat{\alpha}_2$ 为 $\alpha_2 = \frac{1}{m}$ 的估计, $\hat{\rho}_2$ 为 $\rho_2 = \bar{\sigma} + \bar{d}_0$ 的估计, γ_{21} 、 γ_{22} 、 γ_{23} 、 γ_{24} 、 η_{21} 、 η_{22} 、 η_{23} 、 η_{24} 均为大于零的常数.

证明 利用 $\hat{f}(\hat{\mathbf{x}} | \hat{\boldsymbol{\theta}}_f)$ 和 $\hat{g}(\hat{\mathbf{x}} | \hat{\boldsymbol{\theta}}_g)$ 来分别逼近式(22)中未知函数 $f(\mathbf{x})$ 和 $g(\mathbf{x})$, 同时将式(23)、(24)代入式(22)中整理可得

$$\begin{aligned} \dot{\tilde{\mathbf{e}}} = & (\mathbf{A} - \mathbf{K}_c\mathbf{C}^T) \tilde{\mathbf{e}} + \mathbf{B}[-\tilde{\boldsymbol{\theta}}_f^T \xi(\hat{\mathbf{x}}) - \\ & \tilde{\boldsymbol{\theta}}_g^T \xi(\hat{\mathbf{x}})mu + \hat{\boldsymbol{\theta}}_g^T \xi(\hat{\mathbf{x}})m\hat{\alpha}_2 v_2 - \hat{\rho}_2]. \end{aligned} \quad (29)$$

其中: $\hat{\alpha}_2$ 、 $\hat{\rho}_2$ 分别为 α_2 、 ρ_2 估计误差, 定义为 $\hat{\alpha}_2 = \alpha_2 - \hat{\alpha}_2$, $\hat{\rho}_2 = \rho_2 - \hat{\rho}_2$.

构造如下 Lyapunov 候选函数

$$\begin{aligned} V_1 = & \frac{1}{2} \tilde{\mathbf{e}}^T \mathbf{P}_1 \tilde{\mathbf{e}} + \frac{1}{2} \hat{\mathbf{e}}^T \mathbf{P}_2 \hat{\mathbf{e}} + \frac{1}{2\gamma_{21}} \tilde{\boldsymbol{\theta}}_f^T \tilde{\boldsymbol{\theta}}_f + \\ & \frac{m}{2\gamma_{22}} \tilde{\boldsymbol{\theta}}_g^T \tilde{\boldsymbol{\theta}}_g + \frac{m}{2\gamma_{23}} \hat{\alpha}_2^2 + \frac{1}{2\gamma_{24}} \hat{\rho}_2^2. \end{aligned} \quad (30)$$

求 V_1 关于时间的导数

$$\begin{aligned} \dot{V}_1 = & \frac{1}{2} \tilde{\mathbf{e}}^T [(\mathbf{A} - \mathbf{K}_c\mathbf{C}^T)^T \mathbf{P}_1 + \mathbf{P}_1(\mathbf{A} - \mathbf{K}_c\mathbf{C}^T)] \tilde{\mathbf{e}} + \\ & \frac{1}{2} \hat{\mathbf{e}}^T [(\mathbf{A} - \mathbf{B}\mathbf{K}_0^T)^T \mathbf{P}_2 + \mathbf{P}_2(\mathbf{A} - \mathbf{B}\mathbf{K}_0^T)] \hat{\mathbf{e}} + \\ & \hat{\mathbf{e}}^T \mathbf{P}_1 \mathbf{B}[-\tilde{\boldsymbol{\theta}}_f^T \xi(\hat{\mathbf{x}}) - \tilde{\boldsymbol{\theta}}_g^T \xi(\hat{\mathbf{x}})mu + \\ & \hat{\boldsymbol{\theta}}_g^T \xi(\hat{\mathbf{x}})m\hat{\alpha}_2 v_2 - \hat{\rho}_2] + \hat{\mathbf{e}}^T \mathbf{P}_2 \mathbf{K}_c \mathbf{C}^T \tilde{\mathbf{e}} - \frac{1}{\gamma_{21}} \tilde{\boldsymbol{\theta}}_f^T \dot{\tilde{\boldsymbol{\theta}}}_f - \\ & \frac{m}{\gamma_{22}} \tilde{\boldsymbol{\theta}}_g^T \dot{\tilde{\boldsymbol{\theta}}}_g - \frac{m}{\gamma_{23}} \hat{\alpha}_2 \dot{\hat{\alpha}}_2 - \frac{1}{\gamma_{24}} \hat{\rho}_2 \dot{\hat{\rho}}_2. \end{aligned} \quad (31)$$

将式(20)、(21)代入式(31)有

$$\begin{aligned} \dot{V}_1 \leq & -\frac{1}{2} \tilde{\mathbf{e}}^T \mathbf{Q}_1 \tilde{\mathbf{e}} - \frac{1}{2} \hat{\mathbf{e}}^T \mathbf{Q}_2 \hat{\mathbf{e}} + 4 \|\tilde{\mathbf{e}}\|^2 + \\ & \tilde{\boldsymbol{\theta}}_f^T \tilde{\boldsymbol{\theta}}_f \|\mathbf{P}_1\|^2 + \tilde{\boldsymbol{\theta}}_g^T \tilde{\boldsymbol{\theta}}_g \|\mathbf{P}_1\|^2 m^2 u^2 + \\ & \hat{\alpha}_2^2 \|\mathbf{P}_1\|^2 \|\hat{\boldsymbol{\theta}}_g\|^2 m^2 v_2^2 + \hat{\rho}_2^2 \|\mathbf{P}_1\|^2 - \\ & \frac{1}{\gamma_{21}} \tilde{\boldsymbol{\theta}}_f^T \dot{\tilde{\boldsymbol{\theta}}}_f - \frac{m}{\gamma_{22}} \tilde{\boldsymbol{\theta}}_g^T \dot{\tilde{\boldsymbol{\theta}}}_g - \frac{m}{\gamma_{23}} \hat{\alpha}_2 \dot{\hat{\alpha}}_2 - \\ & \frac{1}{\gamma_{24}} \hat{\rho}_2 \dot{\hat{\rho}}_2. \end{aligned}$$

整理可得

$$\begin{aligned} \dot{V}_1 \leq & -\frac{1}{2}\tilde{\mathbf{e}}^T \mathbf{Q}_1 \tilde{\mathbf{e}} - \frac{1}{2}\tilde{\mathbf{e}}^T \mathbf{Q}_2 \tilde{\mathbf{e}} + 4 \|\tilde{\mathbf{e}}\|^2 + \\ & \frac{1}{2}\tilde{\boldsymbol{\theta}}_f^T \tilde{\boldsymbol{\theta}}_f \|\mathbf{P}_1\|^2 + \frac{1}{2}\boldsymbol{\theta}_f^{*T} \boldsymbol{\theta}_f^* \|\mathbf{P}_1\|^2 + \\ & \frac{1}{2}\tilde{\boldsymbol{\theta}}_g^T \tilde{\boldsymbol{\theta}}_g \|\mathbf{P}_1\|^2 m^2 u^2 + \frac{1}{2}\boldsymbol{\theta}_g^{*T} \boldsymbol{\theta}_g^* \|\mathbf{P}_1\|^2 m^2 u^2 + \\ & \frac{1}{2}\tilde{\alpha}_2^2 \|\mathbf{P}_1\|^2 \|\hat{\boldsymbol{\theta}}_g\|^2 m^2 v_2^2 + \frac{1}{2}\|\mathbf{P}_1\|^2 \|\hat{\boldsymbol{\theta}}_g\|^2 v_2^2 + \\ & \frac{1}{2}\tilde{\rho}_2^2 \|\mathbf{P}_1\|^2 + \frac{1}{2}\rho_2^2 \|\mathbf{P}_1\|^2 - \frac{1}{\gamma_{21}}\dot{\tilde{\boldsymbol{\theta}}}_f^T (\dot{\boldsymbol{\theta}}_f + \\ & \gamma_{21}\hat{\boldsymbol{\theta}}_f \|\mathbf{P}_1\|^2) - \frac{m^2}{\gamma_{22}}\dot{\tilde{\boldsymbol{\theta}}}_g^T (\dot{\boldsymbol{\theta}}_g + \gamma_{22}\hat{\boldsymbol{\theta}}_g \|\mathbf{P}_1\|^2 u^2) - \\ & \frac{m^2}{\gamma_{23}}\dot{\tilde{\alpha}}_2^T (\dot{\alpha}_2 + \gamma_{23}\hat{\alpha}_2 \|\mathbf{P}_1\|^2 \|\hat{\boldsymbol{\theta}}_g\|^2 v_2^2) - \frac{1}{\gamma_{24}}\dot{\tilde{\rho}}_2^T (\dot{\rho}_2 + \\ & \gamma_{24}\hat{\rho}_2 \|\mathbf{P}_1\|^2). \end{aligned} \quad (32)$$

假设存在常数 M, N, ϑ 分别满足不等式 $|u| \leq M, |v| \leq N, \|\hat{\boldsymbol{\theta}}_g\|^2 \leq \vartheta$. 将自适应律式 (25) ~ (28) 代入式 (32) 整理有

$$\begin{aligned} \dot{V}_1 \leq & -\frac{1}{2}(\lambda_{\min}(\mathbf{Q}_1) - 8) \|\tilde{\mathbf{e}}\|^2 - \frac{1}{2}\tilde{\mathbf{e}}^T \mathbf{Q}_2 \tilde{\mathbf{e}} - \\ & \tau_1 \tilde{\boldsymbol{\theta}}_f^T \tilde{\boldsymbol{\theta}}_f - \tau_2 m^2 \tilde{\boldsymbol{\theta}}_g^T \tilde{\boldsymbol{\theta}}_g - \tau_3 m^2 \tilde{\alpha}_2^2 - \tau_4 \tilde{\rho}_2^2 + C_1 \leq \\ & -\mu_1 V_1 + C_1. \end{aligned} \quad (33)$$

其中: $\tau_1 = \frac{\eta_{21}}{2\gamma_{21}} - \frac{1}{2}\|\mathbf{P}_1\|^2, \tau_2 = \frac{\eta_{22}}{2\gamma_{22}} - \frac{1}{2}\|\mathbf{P}_1\|^2 M^2,$

$$\tau_3 = \frac{\eta_{23}}{2\gamma_{23}} - \frac{1}{2}\|\mathbf{P}_1\|^2 \vartheta N_2^2, \tau_4 = \frac{\eta_{24}}{2\gamma_{24}} - \frac{1}{2}\|\mathbf{P}_1\|^2,$$

$$\mu_1 = \min \left\{ \frac{\lambda_{\min}(\mathbf{Q}_1) - 8\lambda_{\min}(\mathbf{Q}_2)}{\lambda_{\max}(\mathbf{P}_1)}, \frac{\lambda_{\min}(\mathbf{Q}_2)}{\lambda_{\max}(\mathbf{P}_2)}, \tau_1 \gamma_{21}, \tau_2 \gamma_{22}, \tau_3 \gamma_{23}, \tau_4 \gamma_{24} \right\},$$

$$\begin{aligned} C_1 = & \left(\frac{1}{2}\|\mathbf{P}_1\|^2 + \frac{\eta_{21}}{2\gamma_{21}} \right) \boldsymbol{\theta}_f^{*T} \boldsymbol{\theta}_f^* + \frac{1}{2}\|\mathbf{P}_1\|^2 \vartheta N_2^2 + \\ & \left(\frac{\eta_{22}}{2\gamma_{22}} + \frac{1}{2}\|\mathbf{P}_1\|^2 M^2 \right) m^2 \boldsymbol{\theta}_g^{*T} \boldsymbol{\theta}_g^* + \frac{\eta_{23}}{2\gamma_{23}} + \\ & \left(\frac{1}{2}\|\mathbf{P}_1\|^2 + \frac{\eta_{24}}{2\gamma_{24}} \right) \rho_2^2. \end{aligned}$$

对式 (33) 两端求积分有

$$V_1(t) \leq \left[V_1(0) - \frac{C_1}{\mu_1} \right] e^{-\mu_1 t} + \frac{C_1}{\mu_1}. \quad (34)$$

根据式 (34) 可得存在 T 使得当 $t \geq T$ 时满足

$$e \leq \sqrt{\frac{2C_0}{\lambda_{\min}(\mathbf{P}_1)\mu_0}} + \sqrt{\frac{2C_0}{\lambda_{\min}(\mathbf{P}_2)\mu_0}} \text{ 成立, 选取合}$$

适的参数即可使得跟踪误差任意小, 即实现了跟踪任务, 同时根据 $V(t)$ 的定义可知闭环系统所有信号均有界. 定理 2 证毕.

3 仿真结果

考虑如下非仿射非线性系统

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_1^2 + 0.15D^3(u) + \sin[0.1D(u)] + \\ \quad 0.1(1 + x_2^2)D(u). \end{cases}$$

仿真目的是验证所设计控制器能使系统的输出 y 跟踪期望轨迹 $y_d = \sin(t)$. 仿真实验将分系统状态可测与不可测两种情况进行. 其中死区 $D(u)$ 定义为

$$D(u) = \begin{cases} 0.7(u - 1), & u \geq 1; \\ 0, & -3 < u < 1; \\ u + 3, & u \leq -3. \end{cases}$$

模糊隶属度函数选取分别为

$$\mu_{F_1^1}(x_i) = \frac{1}{1 + \exp[5(x_i + 3)]};$$

$$\mu_{F_1^3}(x_i) = \exp[-(x_i + 1)^2];$$

$$\mu_{F_1^4}(x_i) = \exp(-x_i^2);$$

$$\mu_{F_1^5}(x_i) = \exp[-(x_i - 1)^2];$$

$$\mu_{F_1^6}(x_i) = \exp[-(x_i - 2)^2];$$

$$\mu_{F_1^7}(x_i) = \frac{1}{1 + \exp[-5(x_i - 3)]}, i = 1, 2.$$

状态可测时, 参数取值: $\gamma_1 = 0.01, \gamma_2 = 0.01, \gamma_3 = 0.01, \gamma_4 = 0.01, \eta_1 = 0.08, \eta_2 = 0.08, \eta_3 = 0.08, \eta_4 = 0.08, \mathbf{K} = [25 \ 10]^T, \mathbf{P} = \begin{bmatrix} 2.75 & 0.2 \\ 0.2 & 0.07 \end{bmatrix}$. 仿真过程中初值设定: $\alpha(0) = 0.1, \hat{\boldsymbol{\theta}}_f^T(0) = [0.1 \ \cdots \ 0.1]_{1 \times 49}, x_1(0) = 1, \rho(0) = 0.1, \hat{\boldsymbol{\theta}}_g^T(0) = [0.1 \ \cdots \ 0.1]_{1 \times 49}, x_2(0) = 0$. 仿真时间为 30 s, 仿真结果如图 1 ~ 3 所示.

状态不可测时, 参数取值: $\gamma_{21} = 0.01, \gamma_{22} = 0.01, \gamma_{23} = 0.01, \gamma_{24} = 0.01, \eta_{21} = 0.001, \eta_{22} = 0.001, \eta_{23} = 0.001, \eta_{24} = 0.001, \mathbf{K}_0 = [10 \ 4]^T, \mathbf{K}_c = [4 \ 5]^T, \mathbf{P}_1 = \begin{bmatrix} 1.5 & 1 \\ 1 & 11.5 \end{bmatrix}$. 仿真初值设定: $\alpha(0) = 0.1, \rho(0) = 0.1, \hat{\boldsymbol{\theta}}_f^T(0) = [0.1 \ \cdots \ 0.1]_{1 \times 49}, x_1(0) = 2, x_2(0) = 0, \hat{\boldsymbol{\theta}}_g^T(0) = [0.1 \ \cdots \ 0.1]_{1 \times 49}, \hat{\mathbf{e}}(0) = [1 \ 0]^T$. 仿真时间取为 30 s, 结果如图 4 ~ 6 所示.

由图 1 和图 4 可以观察到系统输出跟踪期望信号的轨迹, 结合跟踪误差图 2 和图 5, 可以得出结论, 利用本文所设计的控制器, 可以使得系统完成跟踪任务, 即跟踪误差收敛于零的某一邻域. 图 3 和图 6 则分别给出了两种情况下系统控制输入

的轨迹曲线.经上述分析,本文给出的控制器能够使得系统输出跟踪给定轨迹,并同时保证了跟踪误差收敛以及控制输入有界.

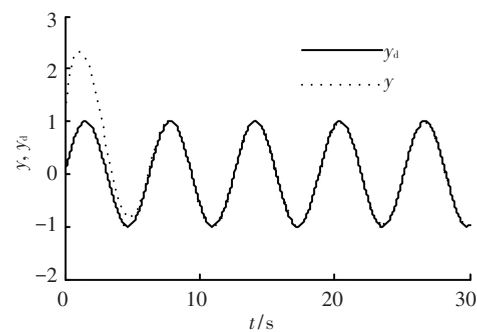


图 1 状态可测时期望输出与实际跟踪曲线

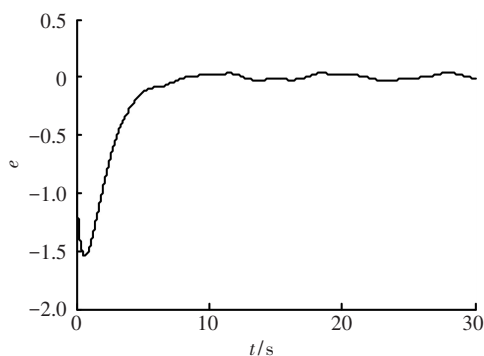


图 2 状态可测时跟踪误差曲线

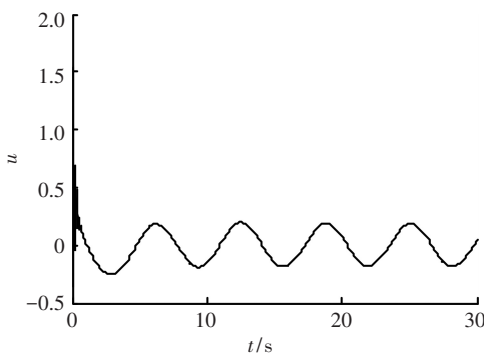


图 3 状态可测时系统控制输入曲线

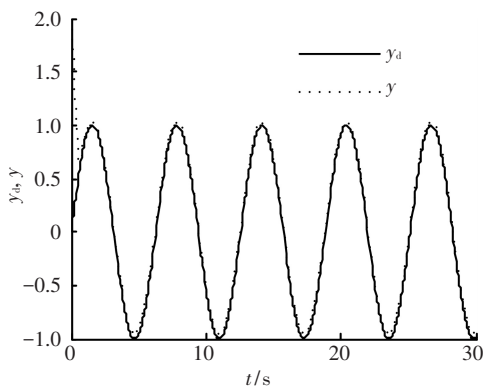


图 4 状态不可测时期望输出与实际跟踪曲线

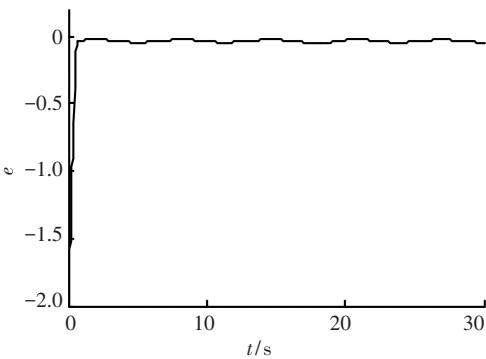


图 5 状态不可测时跟踪误差曲线

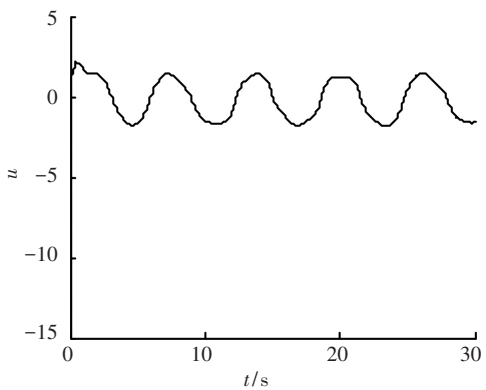


图 6 状态不可测时系统控制输入曲线

5 结 论

- 1) 研究了一类含有未知死区输入环节的 SISO 非仿射非线性系统控制器设计问题,给出了在系统状态可测与不可测两种情况下的控制器设计方案.
- 2) 通过 Lyapunov 定理证明了具有未知死区的 SISO 非仿射非线性闭环系统的稳定性,跟踪误差收敛,同时保证了所有系统均有界.

参考文献

[1] CHO H, BAI E. Convergence results for an adaptive dead zone inverse [J]. Int J Adapt Control Signal Processing, 1998, 12(5): 451-466.

[2] TAWARE A, TAO Gang, TEOLIS C. Design and analysis of a hybrid control scheme for sandwich nonsmooth nonlinear systems [J]. IEEE Trans Autom Control, 2002, 47(1): 145-150.

[3] ZHOU Jing, WEN Changyun, ZHANG Ying. Adaptive output control of nonlinear systems with uncertain dead-zone nonlinearity [J]. IEEE Trans Autom Control, 2006, 51(3): 504-511.

[4] ZHOU Jing, SHEN X Z. Robust adaptive control of nonlinear uncertain plants with unknown dead-zone [J]. IET Control Theory Appl, 2007, 1(1): 25-32.

- [5] LEWIS F L, TIM W K, WANG L Z, et al. Dead-zone compensation in motion control systems using adaptive fuzzy logic control [J]. IEEE Trans Contr Syst Technol, 1999, 7(6): 731-742.
- [6] JANG J O. A dead-zone compensator of a DC motor system using fuzzy logic control [J]. IEEE Trans Man, Systems and Cybernetics: Applications and Reviews, 2001, 31(1): 42-48.
- [7] WANG Xingsong, SU Chunyi, HONG H. Robust adaptive control of a class of linear systems with unknown dead-zone [J]. Automatica, 2004, 40(3): 407-413.
- [8] 张天平, 裔扬, 梅建东. 带有未知死区模型的鲁棒自适应模糊控制[J]. 控制与决策, 2006, 21(4): 367-370.
- [9] IBRIR S, XIE Wenfang, SU Chunyi. Adaptive tracking of nonlinear systems with non-symmetric dead-zone input [J]. Automatica, 2007, 43(3): 522-530.
- [10] 沈启坤, 张天平. 具有未知非线性死区的自适应模糊控制[J]. 控制与决策, 2007, 22(6): 689-692.
- [11] HUA Changchun, WANG Qingguo, GUAN Xinping. Adaptive tracking controller design of nonlinear systems with time-delays and unknown dead-zone input [J]. IEEE Trans Autom Control, 2008, 53(7): 1753-1759.
- [12] ADLOO H, NOROOZI N, KARIMAGHA P. Observer-based model reference adaptive control for unknown time-delay chaotic systems with input nonlinearity [J]. Nonlinear Dynamic, 2012, 67: 1337-1356.
- [13] TONG Shaocheng, LI Yongming. Adaptive fuzzy output feedback control of MIMO nonlinear systems with unknown dead-zone inputs [J]. IEEE Trans Fuzzy Syst, 2013, 21(1): 134-146.
- [14] LABIOD S, GUERRA T M. Adaptive fuzzy control of a class of SISO nonaffine nonlinear systems [J]. Fuzzy Sets Syst, 2007, 158(10): 1126-1137.
- [15] WEN Jie, JIANG Changsheng. Adaptive fuzzy controller for a class of strict-feedback nonaffine nonlinear systems [J]. Journal of Systems Engineering and Electronics, 2011, 22(6): 967-974.
- [16] MAO Zhizhong, XIAO Xiaoshi. Decentralized adaptive tracking control of nonaffine nonlinear large-scale systems with time-delays [J]. Inf Sci, 2011, 181(23): 5291-5303.
- [17] BOULKROUNE A, MSAAD M, FARZA M. Adaptive fuzzy tracking control for a class of MIMO nonaffine uncertain systems [J]. Neurocomputing, 2012, 93: 48-55.
- [18] WANG Lixin. A course in fuzzy systems and control [M]. Englewood Cliffs, NJ: Prentice Hall, 1997.

(编辑 魏希柱)

(上接第 109 页)

- [7] HUANG Z, GEDALANGA P B, ASYPATHANAGUL P, et al. Influence of physicochemical and operational parameters on nitrobacter and nitrospira communities in an aerobic activated sludge bioreactor [J]. Water Research, 2010, 44(15): 4351-4358.
- [8] 曲志军, 尤作亮, 徐洪福. 原水水质在管道输送中的变化规律[J]. 供水技术, 2007, 1(2): 5-9.
- [9] 赵乐乐, 李星, 杨艳玲, 等. 给水管网中硝化作用的形成特性研究[J]. 中国给水排水, 2011, 27(9): 37-40.
- [10] 廖亮. 给水处理中亚硝酸盐来源分析及控制[D]. 西安: 西安建筑科技大学, 2008.
- [11] 张达, 杨艳玲, 李星, 等. 原水水质对输水管道硝化作用形成的影响[J]. 中国环境科学, 2014, 34(1): 2220-2227.
- [12] 王军. 氯胺消毒供水管网硝化作用及其控制措施研究型[D]. 长沙: 湖南大学, 2009.
- [13] 林星志, 杨慧芬, 宋存义. UV_{254} 在水质监测中的应用的研究[J]. 能源与环境, 2006(1): 22-24.
- [14] 蒋绍阶, 刘宗源. UV_{254} 作为水处理中有机物控制指标的意义[J]. 重庆建筑大学学报, 2002, 24(2): 61-65.
- [15] 白晓慧, 张晓红, 张玲, 等. 上海市供水管网中异养菌生长水平及相关指标变化[J]. 环境科学, 2006, 27(11): 2350-2352.
- [16] 桑军强, 余国忠, 王占生. 磷含量与饮用水生物稳定性的关系[J]. 中国环境科学, 2002, 22(6): 534-536.
- [17] 张华星, 陈银广, 杨海真. 增加污水中挥发性脂肪酸以提高生物除磷效果的研究进展[J]. 治理技术, 2005, 24(4): 22-25.
- [18] 刘智晓, 崔福义, 王树, 等. 生物膜工艺实现高效除磷的关键问题及技术线路[J]. 给水排水, 2009, 35(4): 44-49.

(编辑 刘 彤)