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基于余弦速度场的厚板轧制力能参数建模

邓 磊, 章顺虎

(苏州大学 沙钢钢铁学院, 江苏 苏州 215021)

摘 要: 为建立一个预测精度可靠的轧制力模型, 根据轧件变形时金属流动的特点提出了一个余弦速度场, 并基于该速度场进行了相应的能量解析。该速度场能够严格满足体积不变条件、出入口速度边界条件以及几何方程, 较好地满足了运动许可条件。建模过程中, 基于分矢量内积加和法导出了轧制内部变形功率, 解决了非线性 Mises 比塑性功率带来的积分问题。同时, 基于该速度场也导出了摩擦功率、剪切功率的数学表达式。在此基础之上, 依据刚塑性变分原理获得了轧制力能参数的解析模型。利用国内某厂现场轧制数据对建立的轧制力模型进行了验证。结果显示, 各道次轧制力预测值与实测值的误差均在 7.55% 以内, 具有较高的精度。将所建模型与经典的 Sims 模型和 Tselikov 模型进行了对比, 也显示了较好的优越性。此外, 为了研究厚板轧制过程中的参数变化规律, 本文也先后分析了压下率、形状因子、摩擦因子、径厚比、轧辊半径对应力状态系数和中性点位置的影响。

关键词: 厚板; 轧制力; 余弦速度场; 解析模型; 分矢量内积加和法

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Modeling of force-energy parameter of thick plate rolling based on a cosine velocity field

DENG Lei, ZHANG Shunhu

(School of Iron and Steel, Soochow University, Suzhou 215021, Jiangsu, China)

Abstract: To establish a rolling force model with reliable prediction accuracy, a cosine velocity field was proposed according to the characteristics of metal flow in rolling process, and corresponding energy analysis was carried out based on the proposed velocity field. The proposed velocity field could strictly satisfy the volume constant condition, the boundary condition, and the geometric equation, which indicates that the velocity field can satisfy the kinematically admissible condition well. In the modeling process, the inner product and addition method of vector component was adopted to derive the internal deformation power during rolling, and the integration problem of the nonlinear specific plastic work rate of Mises criterion was solved. In addition, the mathematical expressions of friction power and shear power were derived based on the proposed velocity field. The analytical model of force-energy parameters in rolling process was obtained in terms of the variational principle of rigid-plastic material. The prediction accuracy of the rolling force model was verified by using the measured data from a domestic factory. Comparison results show that the deviations between the predicted rolling force and the measured value were within 7.55%, indicating a high accuracy. The established model was compared with the classic Sims model and Tselikov model, and a good superiority was found. In order to investigate the parameter variation during the rolling process of thick plates, the influences of reduction, shape factor, friction factor, radius-thickness ratio, and roll radius on the stress state coefficient and the position of the neutral point were analyzed.

Keywords: thick plate; rolling force; cosine velocity field; analytical model; inner product and addition of vector component

轧制力是进行轧机强度校核与轧制工艺设定及优化的依据。构建预测精准的轧制力模型对于提高板材的尺寸精度和质量有重要的意义。其中, 速度场的设定是进行轧制过程轧件运动及变形分析的基础,

在轧制力模型的构建中处于基础性地位。关于轧制速度场的研究可以追溯到 1975 年 Oh 和 Kobayashi^[1]的工作。他们提出了流函数速度场, 并基于该速度场建立了一个轧制力模型。考虑到速度场设定对轧制力模型精度的影响, Kato 等^[2]随后在 1980 年针对棒材轧制过程中不均匀变形的特点提出了一个加权速度场。这两种速度场的提出很好地促进了轧制力解析建模的发展。

数值模拟是求解轧制力的另一种有效方法。它

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作者简介: 邓 磊(1996—), 男, 硕士研究生

通信作者: 章顺虎, shzhang@suda.edu.cn

由图(1)可知边界条件

$$\begin{cases} x=0, \alpha=\theta; h_x=h_\alpha=h_\theta=h_0, h'_x=-\tan\theta \\ x=l, \alpha=0; h_x=h_\alpha=h_1, h'_x=0 \end{cases} \quad (2)$$

轧件在变形过程中不仅存在着垂直方向的变形,在宽度方向同样存在着变形。但是轧件的宽厚比远大于10,宽展量很小^[20]。图2为轧件的宽展俯视图,其中的宽展假定为抛物线形式,其数学表达式、一阶导数以及平均宽度表达式(3)~(5)。

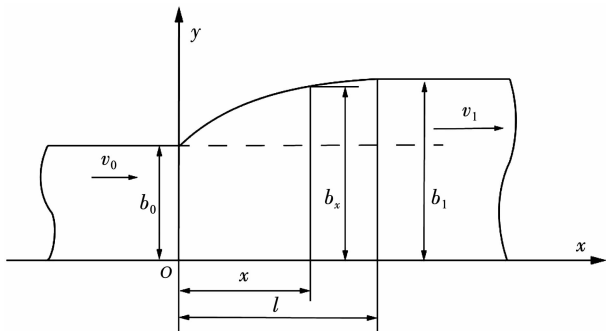


图2 轧制宽度方向变形示意

Fig. 2 Schematic of deformation along width direction of rolling

$$b_x = b_1 - \frac{\Delta b}{l^2}(l-x)^2 \quad (3)$$

$$b'_x = \frac{2\Delta b}{l^2}(l-x) \quad (4)$$

$$\bar{b}_x = \frac{b_0 + 2b_1}{3} \quad (5)$$

1.2 余弦速度场

本文假定轧制时从入口到出口处发生的金属流动符合余弦函数变化规律,写作 $v_x = a - b \cos cx$ 的形式,其表达式以及一、二阶导数为:

$$v_x = \frac{v_0}{2} \left[(1+r) - (r-1) \cos\left(\frac{\pi}{l}x\right) \right] \quad (6)$$

$$v'_x = \frac{v_0 \pi}{2l} (r-1) \sin\left(\frac{\pi}{l}x\right) \quad (7)$$

$$v''_x = \frac{v_0 \pi^2}{2l^2} (r-1) \cos\left(\frac{\pi}{l}x\right) \quad (8)$$

式中 $r = (h_0 b_0) / (h_1 b_1)$ 称为截面压缩比。

根据速度协调条件,可以导出 v_y, v_z 如下:

$$v_y = v_x \frac{h'_x}{h_x} y \quad (9)$$

$$v_z = v_x \frac{h'_x}{h_x} z \quad (10)$$

于是,可得到本文的余弦速度场:

$$\begin{cases} v_x = \frac{v_0}{2} \left[(1+r) - (r-1) \cos\left(\frac{\pi}{l}x\right) \right] \\ v_y = v_x \frac{h'_x}{h_x} y \\ v_z = v_x \frac{h'_x}{h_x} z \end{cases} \quad (11)$$

式中 v_x, v_y, v_z 分别是轧制方向、宽度方向以及压下方向的速度分量。根据几何方程^[21],速度场对应的应变速率场为:

$$\begin{cases} \dot{\epsilon}_x = -v_x \left(\frac{b'_x}{b_x} + \frac{h'_x}{h_x} \right) \\ \dot{\epsilon}_y = v_x \frac{b'_x}{b_x} \\ \dot{\epsilon}_z = v_x \frac{h'_x}{h_x} \\ \dot{\epsilon}_{xy} = -\frac{y}{2} \left(v''_x + \frac{v'_x h'_x + v_x h''_x}{h_x} - \frac{v_x h'^2_x}{h_x^2} \right) \\ \dot{\epsilon}_{xz} = \frac{z}{2h_x} \left(v'_x h'_x + v_x h''_x - \frac{v_x h'^2_x}{h_x} \right) \\ \dot{\epsilon}_{yz} = 0 \end{cases} \quad (12)$$

该速度场能够严格满足体积不变条件、出入口速度边界条件和几何方程,因而可以充分满足运动许可条件。以下部分将以该速度场为基础进行轧制力的能量分析。

2 轧制力的能量解析

2.1 内部变形功率

内部变形功率 N_d 可由变形材料的等效应力和等效应变速率确定,其计算式为

$$N_d = \iiint_V \bar{\sigma} \bar{\dot{\epsilon}} dV = 4 \iiint_V \sigma_s \sqrt{\frac{2}{3} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij}} dV \quad (13)$$

式中: $\bar{\sigma}$ 为轧件等效应力, $\bar{\dot{\epsilon}}$ 为轧件的等效应变速率, σ_s 为变形抗力, $\dot{\epsilon}_{ij}$ 为应变速率分量。

由于式(13)基于的是非线性的 Mises 屈服准则,会给后续的积分计算带来困难,因此使用分矢量内积法和法进行计算,转化过程如下:

$$\begin{aligned} N_d &= 4 \sqrt{\frac{2}{3}} \sigma_s \int_0^{h_x} \int_0^{b_x} \int_0^l \dot{\epsilon} \cdot \dot{\epsilon}_0 dx dy dz = \\ &4 \sqrt{\frac{2}{3}} \sigma_s \left(\iiint_V \eta_x l_x dV + \iiint_V \eta_y l_y dV + \iiint_V \eta_z l_z dV \right) = \\ &4 \sqrt{\frac{2}{3}} \sigma_s (I_x + I_y + I_z) \end{aligned} \quad (14)$$

式中, $\dot{\epsilon} = \eta_x \mathbf{i} + \eta_y \mathbf{j} + \eta_z \mathbf{k}$ 为轧件变形过程中的应变速率矢量, $\dot{\epsilon}_0 = l_x \mathbf{i} + l_y \mathbf{j} + l_z \mathbf{k}$ 为单位向量, $l_x = \cos \alpha$, $l_y = \cos \beta$, $l_z = \cos \gamma$ 分别为单位向量在各个轴上夹角的余弦值。

以下为轧制过程中各个方向的矢量模长以及它们与主轴的夹角余弦。

$$\begin{cases} \eta_x = |\dot{\epsilon}_x| = v_x \left(\frac{b'_x}{b_x} + \frac{h'_x}{h_x} \right) \\ \eta_y = |\dot{\epsilon}_y| + |\dot{\epsilon}_{xy}| = v_x \frac{b'_x}{b_x} + \frac{y}{\sqrt{2}} v_x \left[\frac{b''_x}{b_x} - \frac{h'_x}{h_x} \cdot \frac{b'_x}{b_x} - 2 \left(\frac{b'_x}{b_x} \right)^2 \right] \\ \eta_z = |\dot{\epsilon}_z| + |\dot{\epsilon}_{xz}| = v_x \frac{h'_x}{h_x} + \frac{z}{\sqrt{2}} v_x \left[\frac{h''_x}{h_x} - \frac{h'_x}{h_x} \cdot \frac{b'_x}{b_x} - 2 \left(\frac{h'_x}{h_x} \right)^2 \right] \end{cases} \quad (15)$$

$$\begin{cases} l_x = \cos \alpha = \sqrt{1 + \left(\frac{dy}{dx} \right)^2 + \left(\frac{dz}{dx} \right)^2} \\ l_y = \cos \beta = \sqrt{1 + \left(\frac{dx}{dy} \right)^2 + \left(\frac{dz}{dy} \right)^2} \\ l_z = \cos \gamma = \sqrt{1 + \left(\frac{dx}{dz} \right)^2 + \left(\frac{dy}{dz} \right)^2} \end{cases} \quad (16)$$

注意到式(15)为 x 的单值函数,使用积分中值定理后可得

$$\begin{cases} \frac{h'_x}{h_x} = \frac{1}{l} \int_0^l \frac{h'_x}{h_x} dx = -\frac{\varepsilon_3}{l}; \bar{b}''_x = 0 \\ \bar{h}_x = \frac{h_0 + 2h_1}{3}; \bar{h}''_x = \frac{1}{l} \int_0^l h''_x dx = \frac{1}{R - \Delta h} \\ \bar{h}''_x = \frac{3 \tan \theta}{l(h_0 + 2h_1)} = \frac{3}{(R - \Delta h)(h_0 + 2h_1)} \\ \bar{v}_x = \frac{1}{l} \int_0^l v_x dx = v_0 + \frac{\Delta v}{2}; \bar{v}''_x = \frac{1}{l} \int_0^l v''_x dx = 0 \end{cases} \quad (17)$$

其中: l 为接触弧的水平投影长度, $l = \sqrt{2R\Delta h}$, $\bar{h}_x$ 、 $\bar{b}_x$ 、 $\bar{v}_x$ 分别为按积分中值定理计算所得的变形区平均厚度、平均宽度和轧件沿 x 轴方向平均速度; $\varepsilon_2 = \ln(b_1/b_0)$ 和 $\varepsilon_3 = \ln(h_0/h_1)$ 为轧件宽、厚度方向的主应变。

根据变形区几何关系可以确定:

$$\begin{cases} \frac{dy}{dx} = b'_x \approx \frac{\Delta b}{l} = \bar{b}'_x \\ \frac{dz}{dx} = h'_x = -\frac{\Delta h}{l} = \bar{h}'_x \\ \frac{dz}{dy} = 0 \end{cases} \quad (18)$$

由此可将各方向的应变速率与主轴的余弦值表示成

$$\begin{cases} l_x = \frac{l}{\sqrt{l^2 + \Delta b^2 + \Delta h^2}} \\ l_y = \frac{\Delta b}{\sqrt{\Delta b^2 + l^2}} \\ l_z = \frac{\Delta h}{\sqrt{\Delta h^2 + l^2}} \end{cases} \quad (19)$$

将式(15)~(17)以及积分中值得到的各参数

简化关系代入式(14)后可得到:

$$I_x = \int_0^{h_x} \int_0^{b_x} \int_0^l \frac{v_x \left(\frac{b'_x}{b_x} + \frac{h'_x}{h_x} \right)}{\sqrt{1 + (b'_x)^2 + (h'_x)^2}} dx dy dz = \frac{lU \ln \frac{b_1 h_0}{b_0 h_1}}{\sqrt{l^2 + \Delta b^2 + \Delta h^2}} = Uf_1 \quad (20)$$

$$\begin{aligned} I_y = \int_0^{h_x} \int_0^{b_x} \int_0^l \frac{v_x \frac{b'_x}{b_x} + \frac{y}{\sqrt{2}} v_x \left[\frac{b''_x}{b_x} - \frac{h'_x}{h_x} \cdot \frac{b'_x}{b_x} - 2 \left(\frac{b'_x}{b_x} \right)^2 \right]}{\sqrt{1 + (1/b'_x)^2}} dx dy dz = \\ \frac{\Delta b}{\sqrt{\Delta b^2 + l^2}} \int_0^{h_x} \int_0^{b_x} \int_0^l v_x \frac{b'_x}{b_x} + \\ \frac{y}{\sqrt{2}} v_x \left[\frac{b''_x}{b_x} - \frac{h'_x}{h_x} \cdot \frac{b'_x}{b_x} - 2 \left(\frac{b'_x}{b_x} \right)^2 \right] dx dy dz = \\ \frac{\Delta b U}{\sqrt{\Delta b^2 + l^2}} \left[\ln \frac{b_1}{b_0} + \frac{(\varepsilon_2 \varepsilon_3 - 2\varepsilon_3^2) \bar{b}_x}{2\sqrt{2}l} \right] = Uf_2 \end{aligned} \quad (21)$$

注意到 y 与 z 无关, $dx/dz = 1/h'_x = -l/2\Delta h$, 有

$$\begin{aligned} I_z = \int_0^{h_x} \int_0^{b_x} \int_0^l \frac{v_x \frac{h'_x}{h_x} + \frac{z}{\sqrt{2}} v_x \left[\frac{h''_x}{h_x} - \frac{h'_x}{h_x} \cdot \frac{b'_x}{b_x} - 2 \left(\frac{h'_x}{h_x} \right)^2 \right]}{\sqrt{1 + (1/h'_x)^2}} dx dy dz = \\ \frac{2\Delta h U}{\sqrt{4\Delta h^2 + l^2}} \left\{ \ln \frac{h_0}{h_1} + \frac{1}{2\sqrt{2}l} \left[\frac{l^2}{(R - \Delta h)} + \right. \right. \\ \left. \left. \frac{(\varepsilon_2 \varepsilon_3 - 2\varepsilon_3^2) \bar{h}_x}{1} \right] \right\} = Uf_3 \end{aligned} \quad (22)$$

将逐项积分结果 I_x 、 I_y 、 I_z 代入式(14)后整理得

$$N_d = 4 \sqrt{\frac{2}{3}} \sigma_s (Uf_1 + Uf_2 + Uf_3) = 4 \sqrt{\frac{2}{3}} \sigma_s U \left(\frac{l \ln \frac{b_1 h_0}{b_0 h_1}}{\sqrt{l^2 + \Delta b^2 + \Delta h^2}} + \frac{\Delta b}{\sqrt{\Delta b^2 + l^2}} \left[\ln \frac{b_1}{b_0} + \frac{(\varepsilon_2 \varepsilon_3 - 2\varepsilon_3^2) \bar{b}_x}{2\sqrt{2}l} \right] + \frac{2\Delta h}{\sqrt{4\Delta h^2 + l^2}} \left\{ \ln \frac{h_0}{h_1} + \frac{1}{2\sqrt{2}l} \left[\frac{l^2}{(R - \Delta h)} + \frac{(\varepsilon_2 \varepsilon_3 - 2\varepsilon_3^2) \bar{h}_x}{1} \right] \right\} \right) \quad (23)$$

2.2 摩擦功率

摩擦功率 N_f 可由摩擦剪应力与速度不连续量的乘积求得, 计算式为

$$N_f = 4 \int_s \tau_f \Delta v_f dS \quad (24)$$

式中 Δv_f 为速度不连续量, τ_f 为摩擦剪应力, 其中 Δv_f 和 dS 的计算式为

$$\begin{cases} \Delta v_f = v_R - v_x \sqrt{1 + h'^2_x} = v_R - v_x \sec \alpha \\ dS = \sqrt{1 + h'^2_x} dx dy = \sec \alpha dx dy \end{cases} \quad (25)$$

同样,将轧件表面速度不连续量与摩擦剪应力表示为矢量的形式

$$\begin{cases} \Delta v_f = \Delta v_x \mathbf{i} + \Delta v_y \mathbf{j} + \Delta v_z \mathbf{k} \\ \tau_f = \tau_{fx} \mathbf{i} + \tau_{fy} \mathbf{j} + \tau_{fz} \mathbf{k} \end{cases} \quad (26)$$

将上式代入式(24)后,即可把摩擦功率写成矢量内积的形式

$$\begin{aligned} N_f &= 4 \int_0^l \int_0^{b_x} \tau_f \cdot \Delta v_f dS = \\ &4 \int_0^l \int_0^{b_x} (\tau_{fx} \Delta v_x + \tau_{fy} \Delta v_y + \tau_{fz} \Delta v_z) \sqrt{1 + h'^2_x} dx dy = \\ &4mk \int_0^l \int_0^{b_x} (\Delta v_x \cos \alpha + \Delta v_y \cos \beta + \Delta v_z \cos \gamma) \sec \alpha dx dy = \\ &4mk \left(\int_0^l \int_0^{b_x} \Delta v_x \cos \alpha \sec \alpha dx dy + \int_0^l \int_0^{b_x} \Delta v_y \cos \beta \sec \alpha dx dy + \right. \\ &\left. \int_0^l \int_0^{b_x} \Delta v_z \cos \gamma \sec \alpha dx dy \right) \end{aligned} \quad (27)$$

根据图 1,轧辊表面各方向的余弦值如下

$$\begin{aligned} \cos \alpha &= \pm \frac{\sqrt{R^2 - (l-x)^2}}{R}; \cos \beta = 0 \\ \cos \gamma &= \sin \alpha = \pm \frac{(l-x)}{R} \end{aligned} \quad (28)$$

将式(28)代入式(27)后进行积分计算,并注意以中性点位置为分界点,中性点两侧的积分上下限要交换,分段进行积分后可得到摩擦功率的表达式

$$N_f = 4mk \left[Rv_R b_1 (\theta - 2\alpha_n) + \frac{RU}{h_m} \ln \frac{\tan^2 \left(\frac{\alpha_n}{2} + \frac{\pi}{4} \right)}{\tan \left(\frac{\theta}{2} + \frac{\pi}{4} \right)} \right] \quad (29)$$

2.3 剪切功率

轧制时在变形区入口与出口处的剪切功耗记为 N_{s_0} 与 N_{s_1} , 以下分别进行计算:

$$N_{s_0} = 4 \int k |\Delta \bar{v}_t| dS =$$

$$\begin{aligned} &4k \int_0^{b_0} \int_0^{h_0} (\bar{v}_y \sqrt{1 + (\bar{v}_z/\bar{v}_y)^2}) dz dy = \\ &\frac{k \tan \theta \Delta b U b_0}{b_1 h_0} \sqrt{1 + \frac{4b_1^2 h_0^2}{\Delta b^2 b_0^2}} \end{aligned} \quad (30)$$

式中存在着如下关系:

$$\Delta \bar{v}_t|_{x=0} = \sqrt{\Delta \bar{v}_y^2 + \Delta \bar{v}_z^2}|_{x=0} = \bar{v}_y \sqrt{1 + (\bar{v}_z/\bar{v}_y)^2}|_{x=0} \quad (31)$$

$$\bar{v}_z|_{x=0} = \frac{1}{h_0} \int_0^{h_0} v_z dz = -\frac{\tan \theta v_0}{2} \quad (32)$$

$$\bar{v}_y|_{x=0} = \frac{1}{b_0} \int_0^{b_0} v_y|_{x=0} dy = \frac{\Delta b v_0 b_0 \tan \theta}{4b_1 h_0} \quad (33)$$

在变形区出口处,由于 $h'_x = b'_x = 0$, 因此出口处的金属流动存在着以下规律

$$\bar{v}_y|_{x=l} = \bar{v}_z|_{x=l} = \Delta v_y|_{x=l} = \Delta v_z|_{x=l} = 0 \quad (34)$$

因此,在轧制出口处剪切功率 $N_{s_1} = 0$, 入口处的剪切功率消耗为轧制过程中的总剪切功率,可表示为

$$N_s = N_{s_0} = \frac{k \tan \theta \Delta b U b_0}{b_1 h_0} \sqrt{1 + \frac{4b_1^2 h_0^2}{\Delta b^2 b_0^2}} \quad (35)$$

2.4 总功泛函最小化

根据式(23)、(29)和(35)可将轧制的总功率泛函 Φ 表达成

$$\begin{aligned} \Phi &= N_d + N_f + N_s = 4 \sqrt{\frac{2}{3}} \sigma_s (Uf_1 + Uf_2 + Uf_3) + \\ &4mk \left[Rv_R b_1 (\theta - 2\alpha_n) + \frac{RU}{h_m} \ln \frac{\tan^2 \left(\frac{\alpha_n}{2} + \frac{\pi}{4} \right)}{\tan \left(\frac{\theta}{2} + \frac{\pi}{4} \right)} \right] + \\ &\frac{k \tan \theta \Delta b U b_0}{b_1 h_0} \sqrt{1 + \frac{4b_1^2 h_0^2}{\Delta b^2 b_0^2}} \end{aligned} \quad (36)$$

式中 U 为轧制变形区内任意变形区的秒体积流量。其表达式与一阶偏导式可写为:

$$\begin{aligned} U &= v_x h_x b_x = \left\{ v_0 + \frac{\Delta v}{2} - \frac{\Delta v}{2} \cos \left[\frac{\pi}{l} (l - R \sin \alpha_n) \right] \right\} \\ &(R + h_1 - R \cos \alpha_n) \left\{ b_1 - \frac{\Delta b}{l^2} [l - (l - R \sin \alpha_n)]^2 \right\} \end{aligned} \quad (37)$$

$$\begin{aligned} \frac{dU}{d\alpha_n} &= (R + h_1 - R \cos \alpha_n) \\ &\left\langle -\frac{\Delta v \pi}{2l} \cos \alpha_n \sin \left[\frac{\pi}{l} (l - R \sin \alpha_n) \right] \right\rangle \\ &\left\{ b_1 - \frac{\Delta b}{l^2} [l - (l - R \sin \alpha_n)]^2 \right\} - \\ &\frac{2\Delta b R}{l^2} \cos \alpha_n (l - R \sin \alpha_n) \rangle + \\ &R \sin \alpha_n \left\{ b_1 - \frac{\Delta b}{l^2} [l - (l - R \sin \alpha_n)]^2 \right\} = N \end{aligned} \quad (38)$$

式(23)、(29)和(35)分别对中性角 α_n 求导可得:

$$\frac{dN_d}{d\alpha_n} = 4 \sqrt{\frac{2}{3}} \sigma_s N (f_1 + f_2 + f_3) \quad (39)$$

$$\frac{dN_f}{d\alpha_n} = 4mk \left\{ \frac{2\Delta v R^2}{l^2} \left[\cos \alpha_n \left(1 + \cos \alpha_n - \frac{1}{\cos \alpha_n + 1} \right) - \tan \alpha_n \right] \right\} \quad (40)$$

$$\frac{dN_s}{d\alpha_n} = \frac{k \tan \theta \Delta b N b_0}{b_1 h_0} \sqrt{1 + \frac{4b_1^2 h_0^2}{\Delta b^2 b_0^2}} \quad (41)$$

对总功率泛函变分有

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\alpha_n}=\frac{\partial N_{\mathrm{d}}}{\partial \alpha_n}+\frac{\partial N_{\mathrm{f}}}{\partial \alpha_n}+\frac{\partial N_{\mathrm{s}}}{\partial \alpha_n}\tag{42}$$

基于式(42)可得到摩擦因子 m 的表达式

$$m=-\frac{\sqrt{\frac{2}{3}}\sigma_{\mathrm{s}}N(f_1+f_2+f_3)+\frac{k\tan\theta\Delta bNb_0}{4b_1h_0}\sqrt{1+\frac{4b_1^2h_0^2}{\Delta b^2b_0^2}}}{4v_1(\sec\alpha_n-\cos2\alpha_n)+v_{\mathrm{R}}(\cos2\alpha_n-1)+k\left[\frac{2\Delta vR^2}{l^2}\left[\cos\alpha_n\left(1+\cos\alpha_n-\frac{1}{\cos\alpha_n+1}\right)-\tan\alpha_n\right]\right]}\tag{43}$$

因此,轧制力矩、轧制力以及应力状态系数的解析模型可以根据以下关系进行计算:

$$M=\frac{R}{2v_{\mathrm{R}}}\Phi_{\min};F=\frac{M}{\chi\sqrt{2R\Delta h}};n_{\sigma}=\frac{\bar{\varepsilon}}{2k}=\frac{F}{4blk}\tag{44}$$

3 实验验证

以下用国内某厂实测轧制数据来验证轧制力模型的预测精度。其中,轧件的尺寸为 250 mm × 2 100 mm × 3 500 mm (高 × 长 × 宽), 轧辊直径为 1 200 mm。在本文中,每道次的实际力臂系数 χ 分别为 0.54、0.54、0.53、0.52 和 0.51。经过第一道次的整形轧制后进入展宽轧制阶段,此时轧件的初始厚度为 235.47 mm。材料为 Q345R 钢,其变形抗力模型为^[22]:

$$\sigma_{\mathrm{s}}=6\,310.7\bar{\varepsilon}^{0.407}\bar{\dot{\varepsilon}}\exp(-2.62\times10^{-3}T-0.669\bar{\varepsilon})\tag{45}$$

$$T=t+273\tag{46}$$

式中: $\bar{\varepsilon}$ 为等效应变, $\bar{\dot{\varepsilon}}$ 为等效应变速率, t 为轧制温度, T 为开尔文温度。

各道次的解析轧制力可由式(44)计算得到,相关的参数以及对比结果见表 1、2。

由表 2 易见,轧制力矩计算值与实测值的最大误差均小于 6.83%, 解析轧制力与实测轧制力的最大误差为 7.55%, 平均误差为 4.77%。各个道次的解析轧制力、轧制力矩与实测值之间的误差均满足工程应用上最大预测误差小于 15% 的要求,说明本文建立的轧制力模型具有较好预测精度。

表 1 相关轧制参数介绍

Tab. 1 Summary of relevant rolling parameters

道次	$v_{\mathrm{R}}/$	$t/$	ε	$h_0/$	$h_1/$	$b_0/$	$b_1/$
($\mathrm{m}\cdot\mathrm{s}^{-1}$)		$^{\circ}\mathrm{C}$		m	m	m	m
2	2.27	1 149.75	0.178	0.117	0.098	2.123	2.127
3	2.60	1 144.72	0.228	0.098	0.078	2.127	2.131
4	3.08	1 141.22	0.314	0.078	0.057	2.131	2.133
5	3.74	1 136.81	0.329	0.057	0.041	2.133	2.135
6	4.40	1 128.85	0.312	0.041	0.030	2.135	2.136

表 2 解析轧制力、力矩与实测结果的比较

Tab. 2 Comparison between analytical results of rolling force and torque with the measured results

道次	实测轧制力矩 $M/$ ($\mathrm{kN}\cdot\mathrm{m}$)	解析轧制力矩 $M_{\mathrm{A}}/$ ($\mathrm{kN}\cdot\mathrm{m}$)	相对误差 $\Delta M/\%$	实测轧制力 F/kN	解析轧制力 $F_{\mathrm{A}}/\mathrm{kN}$	相对误差 $\Delta F/\%$
2	2 252	2 375	5.48	27 406	29 199	6.54
3	2 495	2 666	6.83	30 834	33 163	7.55
4	3 028	3 092	2.12	37 956	38 484	1.39
5	2 915	2 824	-3.11	40 378	39 675	-1.74
6	2 065	1 959	-5.12	36 901	34 445	-6.65

将本文所建立的轧制力预测结果与 Sims 模型^[12]、Tselikov 模型^[20]以及实测数据进行对比,结果如图 3 所示。由图可见,Sims 模型的预测结果偏大,而 Tselikov 模型的预测结果偏小,两个经典模型的最大预测误差均超过 15%。而本文所建立的模型的预测结果与实测轧制力吻合较好,最大误差为 6.83%。

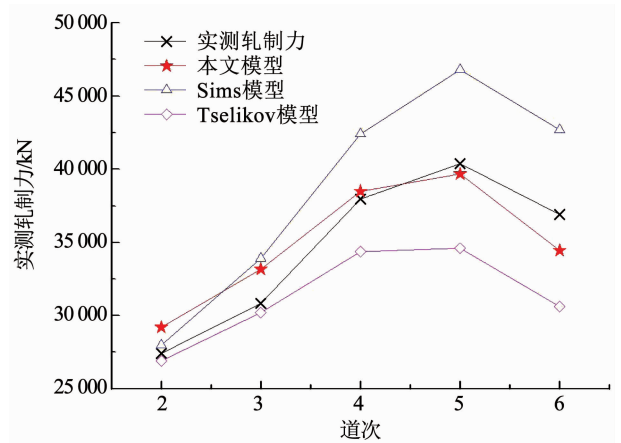


图 3 不同轧制力模型预测效果对比

Fig. 3 Comparison of prediction results of different rolling models

4 分析与讨论

图 4 为不同压下率下各轧制功率的变化关系图。由图可知,在各压下率下,内部变形功率总是轧制过程中消耗能量的最主要部分,且它在总轧制功率中的占比随着压下率的增加而增加,摩擦功率和剪切功率几乎在同一个数量级且摩擦功率数值最小。摩擦功率和剪切功率虽然都与压下率成正相关,但其数值均小于内部变形功率。当摩擦因子为 0.6 时,剪切功率与摩擦功率在压下率为 0.35 时相等,而当摩擦因子为 0.8 时,二者在压下率为 0.3 时相等。此外,随着摩擦因子的增加,摩擦功率的增长速率大于剪切功率的增长速率。因此,随着摩擦因子的增加,摩擦功率等于剪切功率时的所需的压下率会下降。

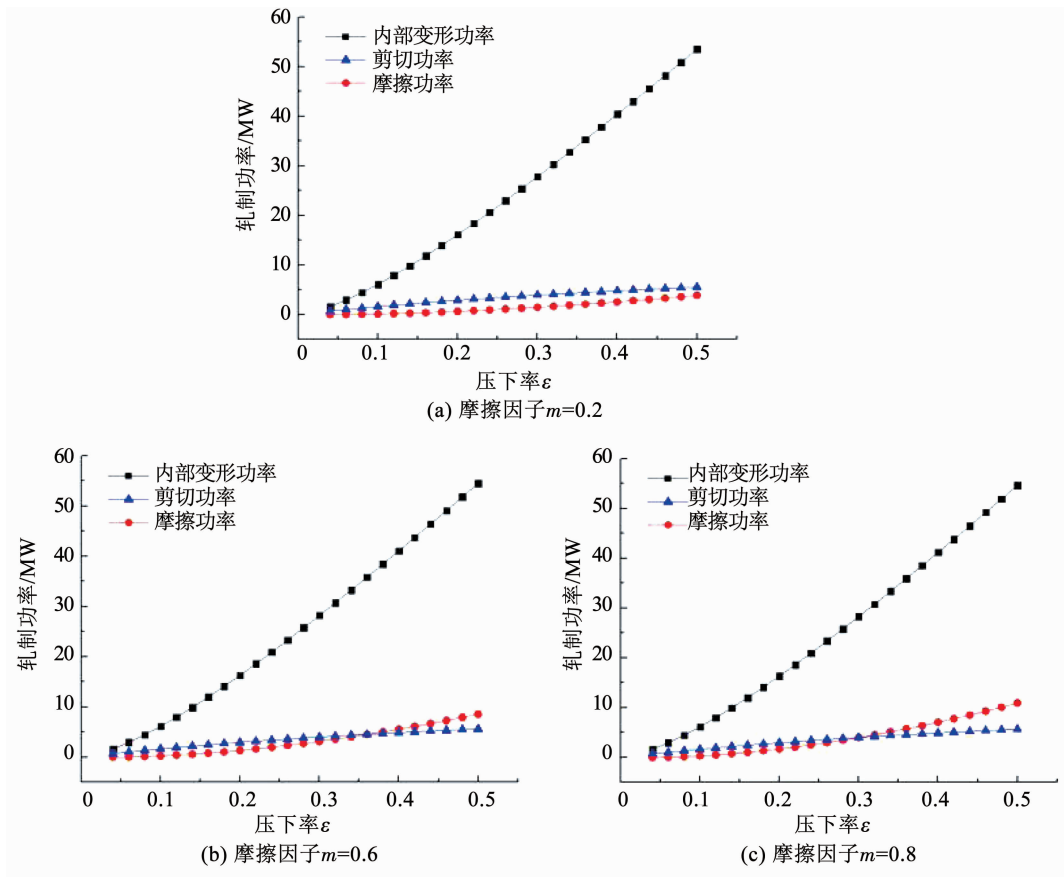


图 4 不同压下率下各轧制功率的变化关系

Fig. 4 Variation of rolling power with different reductions

图 5 为不同压下率下变形区长度 l 与中性点位置 x_n/l 的关系。随着压下率的增大,变形区的总长度虽然变长了,但中性点在变形区内的相对位置并未向出口移动,反而是向变形区中点(图中的中点线)发生了偏移。

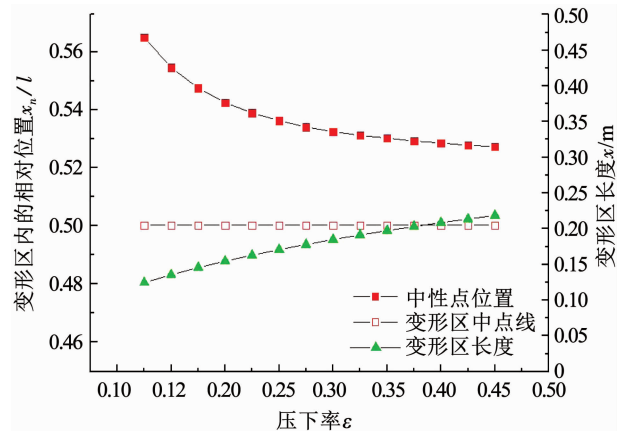


图 5 不同压下率下变形区长度与中性点位置的关系

Fig. 5 Relation between lengths of deformation zone and positions of neutral point under different reductions

图 6 为摩擦因子 m 、压下率 ϵ 与轧制力 F 之间的关系。由图可见,轧制力与摩擦因子以及压下率都成线性正相关。但在压下率较小时,摩擦因子对

轧制力变化的影响较小,而在压下率较大时,摩擦因子对轧制力的影响更明显。

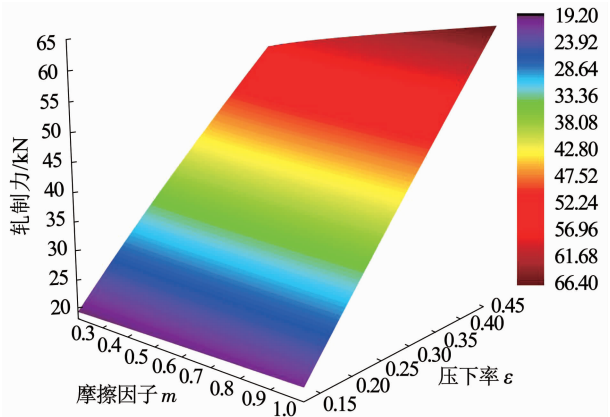


图 6 摩擦因子、压下率与轧制力之间的关系

Fig. 6 Relation among friction factor, reduction, and rolling force

图 7 为压下率 ϵ 、摩擦因子 m 与中性点位置 x_n/l 的关系。图中,摩擦因子对中性点的影响较为显著,而压下率的影响不显著。可以看出,随着摩擦因子的增加,中性点向入口移动。

图 8 为形状因子 $l/2h_m$ 、径厚比 $R/2h_0$ 与应力状态系数 n_σ 的关系。图中,应力状态系数与轧件径厚比成负相关关系,而与形状因子成正相关关系。当

径厚比较大且形状因子较小时,所需的轧制力较小。

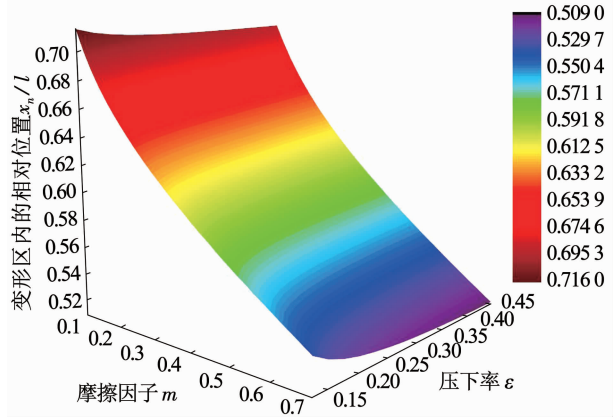


图 7 压下率、摩擦因子对中性点位置的影响

Fig. 7 Influence of reduction and friction factor on positions of neutral point

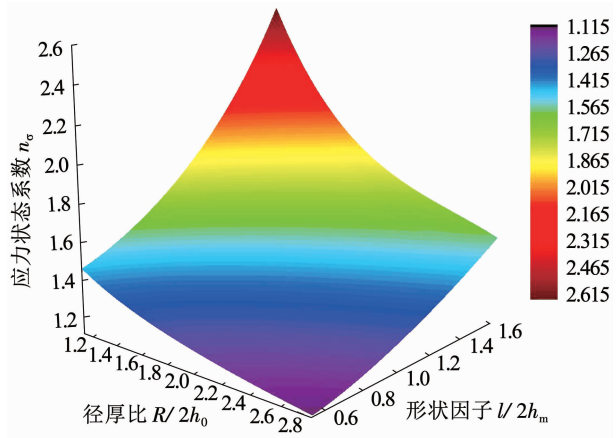


图 8 形状因子、径厚比与应力状态系数的关系

Fig. 8 Relation among shape factor, radius-thickness ratio, and stress state coefficient

5 结 论

1) 本文根据轧件变形区的金属流动特点提出了一个余弦速度场。经证明,该速度场能够严格满足体积不变条件、进出口速度边界条件以及几何方程,这对拟建立的轧制力模型具有一定的精度保障。

2) 针对 Mises 比塑性功率积分困难的问题,提出了一个新的解决方法,即分矢量内积加和法。经此方法得到了轧制的内部变形功率。同时,也基于提出的速度场得到了摩擦功率和剪切功率的表达式,并建立了轧制力的解析模型。本文的理论轧制力矩、轧制力与各道次实测值的误差均在 6.83% 和 7.55% 以内,具有较高精度,表明本文提出的速度场与解析方法是可行的。

3) 参数规律分析表明:在各个压下率下,内部变形功率的消耗都远大于其他两种功率的消耗;随着摩擦因子的增加,摩擦功率的增长速率大于剪切功率的增长速率;轧制中性点的位置并不在变形区

中点,而是在中点偏向出口方向的位置上,但随着压下率的增大,中性点会向变形区中点位置移动。

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