不确定时滞系统的控制器设计

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摘　要: 为了消除时滞和不确定性给实际系统造成的不良影响,采用鲁棒控制系统设计技术,进行鲁棒控制,对具有不确定性的系统,设计1个反馈增益控制器,使系统在不确定性的容许变化范围内满足设计要求,降低系统的灵敏度. 通过构造一种新的 Lyapunov 泛函方法,研究了一类带有时变时滞的不确定系统的鲁棒控制问题. 通过细化不确定信息的结构,给出了基于线性矩阵不等式的控制器设计方法.

关键词: Schur 补引理; 状态反馈控制; 不确定时滞系统

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Controller design for uncertain time-delay systems

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Abstract: To eliminate time delay and uncertainty that cause some undesired effects in the actual system, using robust control system design techniques, a feedback gain controller is designed which allows the system to meet design requirements within the allowable range of uncertainty, and reduces the sensitivity of the system. A class of stability control problem for uncertain time-varying delay systems with state delay is investigated. By materializing uncertain structural information and constructing a new appropriate Lyapunov-Krasovskii functional based on LMIS, the controller design method is derived.

Key words: schur complement lemma; state feedback control; uncertain time-delay systems

1 几个引理[1]

引理1　设 $\boldsymbol{X}$、$\boldsymbol{Y}$ 为向量, 则

$$2\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Y} \leqslant \boldsymbol{X}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{X} + \boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{Y}.$$

其中 $\boldsymbol{Q} > 0$ 对称正定矩阵. 特别地,当 $\boldsymbol{Q} = \varepsilon$ 时,有下面矩阵不等式:

$$2\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Y} \leqslant \varepsilon^{-1}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X} + \varepsilon\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y}$$

成立.

引理2　已知矩阵 $\boldsymbol{E}$、$\boldsymbol{D}$ 和对称矩阵 $\boldsymbol{Y}$, 对任意的不确定矩阵 $\boldsymbol{F}(t)$,如果满足如下矩阵不等式:

$$\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \leqslant \boldsymbol{I},$$

$$\boldsymbol{Y} + \boldsymbol{E}\boldsymbol{F}(t)\boldsymbol{D} + \boldsymbol{D}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{E}^{\mathrm{T}} < 0.$$

则当且仅当存在 $\eta > 0$,使得

$$\boldsymbol{Y} + \eta\boldsymbol{E}\boldsymbol{E}^{\mathrm{T}} + \eta^{-1}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D} < 0.$$

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2 主要结论

考虑下面不确定时变时滞系统:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \boldsymbol{D}\boldsymbol{F}\boldsymbol{E}_1)\boldsymbol{x}(t) + (\boldsymbol{A}_d + \boldsymbol{D}\boldsymbol{F}\boldsymbol{E}_d)\boldsymbol{x}(t - d(t)) + \boldsymbol{B}\boldsymbol{u}(t), & t > 0; \\ \boldsymbol{x}(t) = \phi(t), & t \in [-\tau, 0). \end{cases} \quad (1)$$

其中:$\boldsymbol{x}(t) \in \mathbf{R}^n$ 是系统状态向量;$\boldsymbol{u}(t) \in \mathbf{R}^m$ 是输入向量;$\boldsymbol{A}$ 、$\boldsymbol{A}_d$、$\boldsymbol{B}$ 是已知的适当维数的定常矩阵,$\boldsymbol{A}$ 渐进稳定. $d(t)$ 是时滞可微函数,并且满足下式:

$$0 \leqslant d(t) \leqslant \tau, \quad \dot{d}(t) \leqslant \mu < 1. \quad (2)$$

式中:τ, t 是已知常数, $\varphi(t)$ 是连续向量初始函数. $\boldsymbol{D}$、$\boldsymbol{E}_1$、$\boldsymbol{E}_d$是已知的适当维数定常矩阵,表示不确定性结构信息[2],$\boldsymbol{F}(t) \in \mathbf{R}^{i \times j}$是范数有界的不确定系统模型参数矩阵,其满足

$$\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \leqslant \boldsymbol{I}.$$

假设系统状态可测, 设

$$\boldsymbol{u}(t) = \boldsymbol{K}\boldsymbol{x}(t).$$

其中 $\boldsymbol{K}$ 是适当维数待定控制增益矩阵,则闭环系

统成为

$$\begin{cases} \dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1)\boldsymbol{x}(t) + \\ \qquad (\boldsymbol{A}_d + \boldsymbol{DFE}_d)\boldsymbol{x}(t - d(t)),\ t > 0; \\ \boldsymbol{x}(t) = \phi(t), \qquad t \in [-2\tau, 0). \end{cases}$$

定理 1 对于系统(1),如果存在对称正定矩阵 $\boldsymbol{Q} > 0$, $\tilde{\boldsymbol{P}} > 0$,矩阵 $\tilde{\boldsymbol{K}} > 0$,常数 $\lambda_1 > 0$, $\lambda_2 > 0$,常数 $\lambda_1 > 0$, $\lambda_2 > 0$ 和 $\eta > 0$,满足下列矩阵不等式条件:

$$\begin{bmatrix} \boldsymbol{A}\tilde{\boldsymbol{P}}^{\mathrm{T}} + \boldsymbol{B}\tilde{\boldsymbol{K}} + \boldsymbol{A}_d\tilde{\boldsymbol{P}}^{\mathrm{T}} + \tilde{\boldsymbol{P}}\boldsymbol{A}^{\mathrm{T}} + \tilde{\boldsymbol{K}}\boldsymbol{B}^{\mathrm{T}} + \tilde{\boldsymbol{P}}\boldsymbol{A}_d^{\mathrm{T}} + \boldsymbol{Q} + \tau(\lambda_1 + \lambda_2)\boldsymbol{I} + \eta\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}} & \boldsymbol{A}_d & \boldsymbol{A}_d & \tilde{\boldsymbol{P}}(\boldsymbol{E}_1 + \boldsymbol{E}_d)^{\mathrm{T}} \\ * & -\lambda_1 & 0 & \boldsymbol{E}_d^{\mathrm{T}} \\ * & * & -\lambda_2 & \boldsymbol{E}_d^{\mathrm{T}} \\ * & * & * & -\eta \end{bmatrix} < 0. \tag{3}$$

则系统(1)渐进稳定,且 $\boldsymbol{u}(t) = \boldsymbol{Kx}(t)$ 为其状态反馈控制器,其中 $\boldsymbol{K} = \tilde{\boldsymbol{K}}\tilde{\boldsymbol{P}}^{-\mathrm{T}}$. 上式中“ * ”代表对角位置处矩阵的转置.

证明 由下式

$$\boldsymbol{x}(t - d(t)) = x(t) - \int_{t-d(t)}^{t} \dot{\boldsymbol{x}}(s)\,\mathrm{d}s,$$

有

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1)\boldsymbol{x}(t) + (\boldsymbol{A}_d + \boldsymbol{DFE}_d)(\boldsymbol{x}(t) - \int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s) = (\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1 + \boldsymbol{A}_d + \boldsymbol{DFE}_d)(\boldsymbol{x}(t) - (\boldsymbol{A}_d + \boldsymbol{DFE}_d)\int_{t-d(t)}^{t}((\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1)\boldsymbol{x}(s) + (\boldsymbol{A}_d + \boldsymbol{DFE}_d)\boldsymbol{x}(s - d(s))\mathrm{d}s.$$

其中 $t > 0$.

定义 Lyapunov-Krasovskii 泛函如下:

$$V(\boldsymbol{x}_t) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Px}(t) + \int_{t-d(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{PQP}^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{P}_1\boldsymbol{x}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\int_{t-d(t)+\theta}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{P}_2\boldsymbol{x}(s)\mathrm{d}s\mathrm{d}\theta.$$

其中 $\boldsymbol{P} > 0, \boldsymbol{Q} > 0, \boldsymbol{P}_1 > 0, \boldsymbol{P}_2 > 0$ 是适当维数正定加权矩阵,这样 $\boldsymbol{V}(x_t)$ 就是正定的 Lyapunov-Krasovskii 泛函.

把 $\boldsymbol{V}(\boldsymbol{x}_t)$ 沿着系统(1)的轨迹进行微分,得

$$\dot{\boldsymbol{V}}(\boldsymbol{x}_t)\Big|_{(1)} = \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Px}(t)) + \frac{\mathrm{d}}{\mathrm{d}t}(\int_{t-d(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{PQP}^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s) + \frac{\mathrm{d}}{\mathrm{d}t}(\int_{-\tau}^{0}\int_{t+\theta}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{P}_1\boldsymbol{x}(s)\mathrm{d}s\mathrm{d}\theta) + \frac{\mathrm{d}}{\mathrm{d}t}(\int_{-\tau}^{0}\int_{t-d(t)+\theta}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{P}_2\boldsymbol{x}(s)\mathrm{d}s\mathrm{d}\theta).$$

其中

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Px}(t)) = \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Px}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t) = 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t) = 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\{(\boldsymbol{A} + \boldsymbol{DFE}_1 + \boldsymbol{BK} + \boldsymbol{A}_d + \boldsymbol{DFE}_d)\boldsymbol{x}(t) - (\boldsymbol{A}_d + \boldsymbol{DFE}_d)\int_{t-d(t)}^{t}((\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1)\boldsymbol{x}(s) + (\boldsymbol{A}_d + \boldsymbol{DFE}_d)\boldsymbol{x}(s - d(s))\mathrm{d}s\} = 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1 + X\boldsymbol{DFE}_d)\boldsymbol{x}(t) - 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A}_d + \boldsymbol{DFE}_d)\int_{t-d(t)}^{t}(\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1)x(s)\mathrm{d}s - 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A}_d + \boldsymbol{DFE}_d)\int_{t-d(t)}^{t}(\boldsymbol{A}_d + \boldsymbol{DFE}_d)\boldsymbol{x}(s - d(s))\mathrm{d}s.$$

由引理 1 知,存在常量 $\lambda_1 > 0, \lambda_2 > 0$, 经推导整理,得下式:

$$-2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A}_d + \boldsymbol{DFE}_d)\int_{t-d(t)}^{t}(\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1)\boldsymbol{x}(s)\mathrm{d}s \leqslant \lambda_1^{-1}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A}_d + \boldsymbol{DFE}_d)(\boldsymbol{A}_d + \boldsymbol{DFE}_d)^{\mathrm{T}}\boldsymbol{Px}(t) + \lambda_1\int_{t-d(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1)^{\mathrm{T}}(\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1)\boldsymbol{x}(s)\mathrm{d}s - 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A}_d + \boldsymbol{DFE}_d)\int_{t-d(t)}^{t}(\boldsymbol{A}_d + \boldsymbol{DFE}_d)\boldsymbol{x}(s - d(s))\mathrm{d}s \leqslant \lambda_2^{-1}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A}_d + \boldsymbol{DFE}_d)(\boldsymbol{A}_d + \boldsymbol{DFE}_d)^{\mathrm{T}}\boldsymbol{Px}(t) + \lambda_2\int_{t-d(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s - d(s))(\boldsymbol{A}_d + \boldsymbol{DFE}_d)^{\mathrm{T}}(\boldsymbol{A}_d + \boldsymbol{DFE}_d)\boldsymbol{x}(s - d(s))\mathrm{d}s,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\int_{t-d(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{PQP}^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PQP}^{\mathrm{T}}\boldsymbol{x}(t) - (1 - \dot{d}(t))\boldsymbol{x}^{\mathrm{T}}(t - d(t))\boldsymbol{PQP}^{\mathrm{T}}\boldsymbol{x}(t - d(t)) \leqslant \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PQP}^{\mathrm{T}}\boldsymbol{x}(t) - (1 - \mu)\boldsymbol{x}^{\mathrm{T}}(t - d(t))\boldsymbol{PQP}^{\mathrm{T}}\boldsymbol{x}(t - d(t)),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\int_{-\tau}^{0}\int_{t+\theta}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{P}_1\boldsymbol{x}(s)\mathrm{d}s\mathrm{d}\theta) = \tau\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_1\boldsymbol{x}(t) - \int_{-\tau}^{0}\boldsymbol{x}^{\mathrm{T}}(t + \theta)\boldsymbol{P}_1\boldsymbol{x}(t + \theta)\mathrm{d}\theta,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\int_{-\tau}^{0}\int_{t-d(t)+\theta}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{P}_2\boldsymbol{x}(s)\mathrm{d}s\mathrm{d}\theta) = \tau\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_2\boldsymbol{x}(t) - (1 - \dot{d}(t))\int_{-\tau}^{0}\boldsymbol{x}^{\mathrm{T}}(t - d(t) + \theta)\boldsymbol{P}_2\boldsymbol{x}(t - d(t) + \theta)\mathrm{d}\theta \leqslant \tau\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_2\boldsymbol{x}(t) - (1 - \mu)\int_{-\tau}^{0}\boldsymbol{x}^{\mathrm{T}}(t - d(t) + \theta)\boldsymbol{P}_2\boldsymbol{x}(t - d(t) + \theta)\mathrm{d}\theta.$$

因此

$$\dot{\boldsymbol{V}}(\boldsymbol{x}_t)|_{(1)} \leqslant 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{DFE}_1 + \boldsymbol{A}_d + \boldsymbol{DFE}_d)\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{PQP}^{\mathrm{T}}\boldsymbol{x}(t) - (1 - \mu)\boldsymbol{x}^{\mathrm{T}}(t -$$

$d(t))PQP^{T}x(t-d(t))+\tau x^{T}(t)(P_1+P_2)x(t)+\lambda_1^{-1}x^{T}(t)P(A_d+DFE_d)(A_d+DFE_d)^{T}Px(t)+\lambda_2^{-1}x^{T}(t)P(A_d+DFE_d)(A_d+DFE_d)^{T}Px(t)+\lambda_1\int_{t-d(t)}^{t}x^{T}(s)(A+BK+DFE_1)^{T}(A+BK+DFE_1)x(s)\mathrm{d}s+\lambda_2\int_{t-d(t)}^{t}x^{T}(s-d(s))(A_d+DFE_d)^{T}(A_d+DFE_d)x(s-d(s))\mathrm{d}s-\int_{-\tau}^{0}x^{T}(t+\theta)P_1x(t+\theta)\mathrm{d}\theta-(1-\mu)\int_{-\tau}^{0}x^{T}(t-d(t)+\theta)P_2x(t-d(t)+\theta)\mathrm{d}\theta.$

由文献[3]，存在 $\alpha>0$，$\beta>0$

$$x^{T}(s)(A+BK+DFE_1)^{T}(A+BK+DFE_1)x(s)\leqslant\alpha x^{T}(s)x(s),$$

$$x^{T}(s-d(s))(A_d+DFE_d)^{T}(A_d+DFE_d)x(s-d(s))\leqslant\beta x^{T}(s-d(s))x(s-d(s)).$$

再用文献[4]类似的方法，得

$$\dot V(x_t)\mid_{(1)}\leqslant 2x^{T}(t)P(A+BK+DFE_1+A_d+DFE_d)x(t)+x^{T}(t)PQP^{T}x(t)-(1-\mu)x^{T}(t-d(t))PQP^{T}x(t-d(t))+\tau x^{T}(t)(P_1+P_2)x(t)+\lambda_1^{-1}x^{T}(t)P(A_d+DFE_d)(A_d+DFE_d)^{T}Px(t)+\lambda_2^{-1}x^{T}(t)P(A_d+DFE_d)(A_d+DFE_d)^{T}Px(t)+\lambda_1\int_{t-d(t)}^{t}x^{T}(s)\alpha x(s)\mathrm{d}s+\lambda_2\int_{t-d(t)}^{t}x^{T}(s-d(s))\beta x(s-d(s))\mathrm{d}s-\int_{-\tau}^{0}x^{T}(s)P_1x(s)\mathrm{d}s-(1-\mu)\int_{-\tau}^{0}x^{T}(s-d(s))P_2x(s-d(s))\mathrm{d}s.$$

又由式(2)得

$$-\int_{t-\tau}^{t}x^{T}(s)P_1x(s)\mathrm{d}s=-\left(\int_{t-\tau}^{t-d(t)}x^{T}(s)P_1x(s)\mathrm{d}s+\int_{t-d(t)}^{t}x^{T}(s)P_1x(s)\mathrm{d}s\right)-(1-\mu)\int_{t-\tau}^{t}x^{T}(s-d(s))P_2x(s-d(s))\mathrm{d}s=-(1-\mu)\left(\int_{t-\tau}^{t-d(t)}x^{T}(s-d(s))P_2x(s-d(s))\mathrm{d}s+(1-\mu)\int_{t-d(t)}^{t}x^{T}(s-d(s))P_2x(s-d(s))\mathrm{d}s\right).$$

取 $P_1=\lambda_1\alpha I, P_2=\lambda_2\beta I$，则整理得

$$\dot V(x_t)\mid_{(1)}\leqslant 2x^{T}(t)P(A+BK+DFE_1+A_d+DFE_d)x(t)+x^{T}(t)PQP^{T}x(t)+\tau x^{T}(t)(P_1+P_2)x(t)+\lambda_1^{-1}x^{T}(t)P(A_d+DFE_d)(A_d+DFE_d)^{T}Px(t)+\lambda_2^{-1}x^{T}(t)P(A_d+DFE_d)(A_d+DFE_d)^{T}Px(t)-(1-\mu)x^{T}(t-d(t))Qx(t-d(t))\triangleq x^{T}(t)M(\varepsilon)x(t)-x^{T}(t-d(t))(1-\mu)Qx(t-d(t)).$$

其中：

$$M=2P(A+BK+DFE_1+A_d+DFE_d)+PQP^{T}+\tau(P_1+P_2)+\lambda_1^{-1}P(A_d+DFE_d)(A_d+DFE_d)^{T}P+\lambda_2^{-1}P(A_d+DFE_d)(A_d+DFE_d)^{T}P.$$

显然，若 $M(\varepsilon)<0$，而

$$-x^{T}(t-d(t))(1-\mu)Qx(t-d(t))<0,$$

则 $\dot V(x_t)\mid_{(1)}<0$，系统渐进稳定.

由假设易知 $-x^{T}(t-d(t))(1-\mu)Qx(t-d(t))<0$ 成立.

做限定[5] $\alpha I\leqslant PP^{T}$、$\beta I\leqslant PP^{T}$，则由合同变换得下式：

$$P^{-1}MP^{-T}=2(A+BK+DFE_1+A_d+DFE_d)P^{-T}+Q+\tau P^{-1}(\lambda_1\alpha+\lambda_2\beta)P^{-T}+\lambda_1^{-1}(A_d+DFE_d)(A_d+DFE_d)^{T}+\lambda_2^{-1}(A_d+DFE_d)(A_d+DFE_d)^{T}\leqslant 2(A+BK+DFE_1+A_d+DFE_d)P^{-T}+Q+\tau P^{-1}(\lambda_1PP^{T}+\lambda_2PP^{T})P^{-T}+\lambda_1^{-1}(A_d+DFE_d)(A_d+DFE_d)^{T}+\lambda_2^{-1}(A_d+DFE_d)(A_d+DFE_d)^{T}=(A+BK+DFE_1+A_d+DFE_d)P^{-T}+P^{-1}(A+BK+DFE_1+A_d+DFE_d)^{T}+Q+\tau(\lambda_1+\lambda_2)+\lambda_1^{-1}(A_d+DFE_d)(A_d+DFE_d)^{T}+\lambda_2^{-1}(A_d+DFE_d)(A_d+DFE_d)^{T}<0.$$

再由 Schur 补引理，$M<0$ 等价于下面矩阵不等式成立：

$$\begin{bmatrix}\Omega & A_d+DFE_d & A_d+DFE_d\\ * & -\lambda_1 & 0\\ * & * & -\lambda_2\end{bmatrix}<0. \quad (4)$$

其中“ * ”号的意义同上，

$$\Omega=(A+BK+DFE_1+A_d+DFE_d)P^{-T}+P^{-1}(A+BK+DFE_1+A_d+DFE_d)^{T}+Q+\tau(\lambda_1+\lambda_2).$$

把式(4)作如下处理，以消除不确定：

$$P^{-1}MP^{-T}=\begin{bmatrix}\Pi & A_d & A_d\\ * & -\lambda_1 & 0\\ * & * & -\lambda_2\end{bmatrix}+\begin{bmatrix}\Sigma & DFE_d & DFE_d\\ * & 0 & 0\\ * & * & 0\end{bmatrix}=\begin{bmatrix}\Pi & A_d & A_d\\ * & -\lambda_1 & 0\\ * & * & -\lambda_2\end{bmatrix}+\begin{bmatrix}D\\0\\0\end{bmatrix}F\left[(E_1+E_d)P^{-T}\quad E_d\quad E_d\right]+\left[(E_1+E_d)P^{-T}\quad E_d\quad E_d\right]^{T}F^{T}\begin{bmatrix}D\\0\\0\end{bmatrix}^{T}.$$

其中：

$$\Pi=(A+BK+A_d)P^{-T}+P^{-1}(A+BK+A_d)^{T}+Q+\tau(\lambda_1+\lambda_2)I,$$

$$\boldsymbol{\Sigma} = (\boldsymbol{DFE}_1 + \boldsymbol{DFE}_d)\boldsymbol{P}^{-\mathrm{T}} + \boldsymbol{P}^{-1}(\boldsymbol{DFE}_1 + \boldsymbol{DFE}_d)^{\mathrm{T}}.$$

由引理2知,存在 $\eta > 0$, 使得 $\boldsymbol{P}^{-1}\boldsymbol{M}\boldsymbol{P}^{-\mathrm{T}} < 0$, 等价于

$$\begin{bmatrix} \boldsymbol{\Pi} & \boldsymbol{A}_d & \boldsymbol{A}_d \\ \boldsymbol{A}_d^{\mathrm{T}} & -\lambda_1 & 0 \\ \boldsymbol{A}_d^{\mathrm{T}} & 0 & -\lambda_2 \end{bmatrix} + \eta \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} [\boldsymbol{D}^{\mathrm{T}} \quad 0 \quad 0] + \eta^{-1} \begin{bmatrix} \boldsymbol{P}^{-1}(\boldsymbol{E}_1 + \boldsymbol{E}_d)^{\mathrm{T}} \\ \boldsymbol{E}_d^{\mathrm{T}} \\ \boldsymbol{E}_d^{\mathrm{T}} \end{bmatrix} [(\boldsymbol{E}_1 + \boldsymbol{E}_d)\boldsymbol{P}^{-\mathrm{T}} \quad \boldsymbol{E}_d \quad \boldsymbol{E}_d] < 0,$$

即

$$\begin{bmatrix} (\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{A}_d)\boldsymbol{P}^{-T} + \boldsymbol{P}^{-1}(\boldsymbol{A} + \boldsymbol{BK} + \boldsymbol{A}_d)^{\mathrm{T}} + \boldsymbol{Q} + \tau(\lambda_1 + \lambda_2)\boldsymbol{I} + \eta\boldsymbol{DD}^{\mathrm{T}} & \boldsymbol{A}_d & \boldsymbol{A}_d & \boldsymbol{P}^{-1}(\boldsymbol{E}_1 + \boldsymbol{E}_d)^{\mathrm{T}} \\ * & -\lambda_1 & 0 & \boldsymbol{E}_d^{\mathrm{T}} \\ * & * & -\lambda_2 & \boldsymbol{E}_d^{\mathrm{T}} \\ * & * & * & -\eta \end{bmatrix} < 0.$$

定义 $\boldsymbol{P}^{-1} = \tilde{\boldsymbol{P}}$、$\boldsymbol{K}\tilde{\boldsymbol{P}}^{\mathrm{T}} = \tilde{\boldsymbol{K}}$,得

$$\begin{bmatrix} \boldsymbol{A}\tilde{\boldsymbol{P}}^{\mathrm{T}} + \boldsymbol{B}\tilde{\boldsymbol{K}} + \boldsymbol{A}_d\tilde{\boldsymbol{P}}^{\mathrm{T}} + \tilde{\boldsymbol{P}}\boldsymbol{A}^{\mathrm{T}} + \tilde{\boldsymbol{K}}\boldsymbol{B}^{\mathrm{T}} + \tilde{\boldsymbol{P}}\boldsymbol{A}_d^{\mathrm{T}} + \boldsymbol{Q} + \tau(\lambda_1 + \lambda_2)\boldsymbol{I} + \eta\boldsymbol{DD}^{\mathrm{T}} & \boldsymbol{A}_d & \boldsymbol{A}_d & \tilde{\boldsymbol{P}}(\boldsymbol{E}_1 + \boldsymbol{E}_d)^{\mathrm{T}} \\ * & -\lambda_1 & 0 & \boldsymbol{E}_d^{\mathrm{T}} \\ * & * & -\lambda_2 & \boldsymbol{E}_d^{\mathrm{T}} \\ * & * & * & -\eta \end{bmatrix} < 0.$$

上式即为条件(3),对于变量 $\tilde{\boldsymbol{P}}$、$\tilde{\boldsymbol{K}}$、$\boldsymbol{Q}$、λ_1、λ_2 和 η 是线性的. 则 $\boldsymbol{u}(t) = \boldsymbol{Kx}(t)$ 就为系统(1)的状态反馈控制器,其中 $\boldsymbol{K} = \tilde{\boldsymbol{K}}\tilde{\boldsymbol{P}}^{-\mathrm{T}}$.

证毕

在系统(1)中,令 $\boldsymbol{E}_1 = 0, \boldsymbol{E}_d = 0$,得到如下矩阵不等式:

$$\begin{bmatrix} \boldsymbol{\Delta} & \boldsymbol{A}_d & \boldsymbol{A}_d & \tilde{\boldsymbol{P}} \\ * & -\lambda_1 & 0 & 0 \\ * & * & -\lambda_2 & 0 \\ * & * & * & -\eta \end{bmatrix} < 0.$$

其中:

$$\boldsymbol{\Delta} = \boldsymbol{A}\tilde{\boldsymbol{P}}^{\mathrm{T}} + \boldsymbol{B}\tilde{\boldsymbol{K}} + \boldsymbol{A}_d\tilde{\boldsymbol{P}}^{\mathrm{T}} + \tilde{\boldsymbol{P}}\boldsymbol{A}^{\mathrm{T}} + \tilde{\boldsymbol{K}}\boldsymbol{B}^{\mathrm{T}} + \tilde{\boldsymbol{P}}\boldsymbol{A}_d^{\mathrm{T}} + \boldsymbol{Q} + \tau(\lambda_1 + \lambda_2)\boldsymbol{I} + \eta\boldsymbol{DD}^{\mathrm{T}}.$$

这正是正常系统的稳定性条件[6].

3 结 论

本文考虑一类带时变时滞的不确定系统,研究系统的鲁棒控制器设计问题.所得控制器设计方法描述为线性矩阵不等式形式,容易利用现有优化方法求解相关问题[5,7]. 与现有文献[8-10]相比,系统不确定性结构更加具体.因此,所提方法易于实现且具有广泛的应用前景.

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